MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXV. DECEMBER 11, 1914. No. 2

F. W. DYSON, Esq., M.A., LL.D., F.R.S., VICE-PRESIDENT, in the Chair.

Dr. Eugène de Krudy, Director of the Flammarion Observatory, Bâle, Switzerland,

was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Harold William Acton, Royal Observatory, Greenwich, and 55 Devonshire Road, Greenwich, S.E. (proposed by F. W. Dyson);

Herbert William Unthank, B.A., King's College, Cambridge (proposed by Arthur Berry);

Mary A. Blagg, Greenhill, Cheadle, Staffordshire (proposed by H. H. Turner); and

Mrs. Fiametta Wilson, Hansol Road, Bexley Heath (proposed by J. R. Leeson).

Forty presents were announced as having been received since the last meeting, and the thanks of the Society were returned to the respective donors.

On the Stability of Direct and Retrograde Satellite Orbits. By F. R. Moulton, Professor of Astronomy in the University of Chicago, and Research Associate of the Carnegie Institution of Washington.

1. The fact that the most remote satellites of Jupiter and Saturn revolve in the retrograde direction suggests the inquiry whether the orbits of direct satellites revolving at their respective distances would not be unstable. A start on the discussion of this subject was made by Mr. John Jackson in the December 1913 number of the *Monthly Notices*. The purpose of this paper is to show how the character of the stability can be determined, at least in the case of periodic orbits, and to prove that a retrograde periodic orbit having the period of J VIII. is very stable. Involved in this discussion is some new theory respecting practical means of determining the character of the periodic orbit as regards its stability.

It was intended to determine the character of the stability of a direct orbit having the same period as J VIII. A periodic orbit of this period was computed, and it was found that its orbit is unstable, with "even instability," using the terminology introduced by Sir George Darwin in his memoir in *Acta Mathematica*, vol. xxi. The even instability of the direct satellite proves that the one found belongs to Darwin's "Class B" of periodic satellite orbits. Hence there is also one of "Class C," and it will be stable with a small margin of stability. The fate of "Satellite A" has not been determined as yet. But these questions, together with that of developing a workable definition of stability for a non-period orbit, are being considered.

2. Since at the present time there are sharp definitions of stability only for periodic orbits, it is necessary to consider those in which the motion is periodic. Mr. Jackson seems to regard the field of retrograde periodic orbits as untouched, except for a single orbit computed by Sir George Darwin. As a matter of fact, the theory of retrograde periodic orbits is on precisely the same basis, except for detailed numerical illustrations, as that of direct periodic orbits. This is true for both satellite and inferior planet types of motion, and also for superior planets where, as yet, no numerical examples have been worked out.

As preliminary to an examination of the question under consideration, the facts regarding the existence of periodic orbits of the satellite and planetary type will be explained. The question of the existence of periodic satellite and inferior planet orbits was considered* by the writer in 1906. Those classes were treated which are the analytic continuation of circular orbits (at the limit of zero dimensions) with respect to a parameter which may be either the ratio of the mean motion of the remoter two bodies to that of the closer two (*e.g.*, in the Lunar Theory, the ratio of the mean motion of the Sun to that of the Moon), or the parameter used by Hill in his Lunar Theory. These orbits in Poincaré's classification

* *Trans. of the Am. Math. Soc.*, vol. vii. (1906), pp. 537–577.

belong to *Solutions de la première sorte*, though they were not treated by him. His orbits reduce to circles with the vanishing of the mass of the perturbing bodies, and hence are not applicable to the Lunar Theory; those treated by the writer reduce to circles as their linear dimensions vanish, regardless of the mass of the perturbing body. In the original paper by the writer all three masses were supposed to be finite. For simplicity in the present discussion it will be supposed that the satellite is infinitesimal. The formulæ of the former paper reduce to this case by putting $M = 0$.

It was shown in the paper cited that there are exactly three families of direct periodic satellite orbits of the analytic type in question, but for small values of the parameter the co-ordinates of two of them are complex. Sir George Darwin's computations, *Acta Mathematica*, vol. xxi., show that the two complex orbits unite and become real. The question arises whether this is a phenomenon peculiar to the distribution of masses which he chose for his numerical experiments. It has been proved * by the writer that such a double periodic orbit can disappear with variation of the distribution of the mass, only by uniting with another such an one. But with only three orbits there cannot exist at the same time another double orbit, and hence there is one double periodic orbit for all distribution of the masses, and in particular for the Sun, Jupiter, and a satellite.

The discussion of direct periodic satellite orbits differs from that of retrograde orbits only in the sign of a parameter. There are, therefore, also exactly three families of retrograde periodic orbits of the analytical type in question, and for small values of the parameter the co-ordinates of two of them are complex. It is not at present known whether or not the complex ones become real; but the question could be answered without great difficulty in a special case by numerical experiment, and the result extended to all distributions of the mass by the application of the theorem on the disappearance of double periodic orbits.

The discussion in the paper cited was not simply existential, because the series which give the co-ordinates were worked out to the ninth power inclusive of the parameter. An application was made to the variational and parallactic terms of the Moon's orbit. A comparison was made with Darwin's "Satellite A" for the Jacobian constant (Darwin's notation) equal to 40·5. The identity of the results establishes the analytical character of his orbits A. A comarison was also made with his "Planets A," and the agreement of the results establishes the analytic character of these orbits. They are of the same type as "Satellites A," and from the analytical point of view there is no reason for distinguishing between satellites and planets. Further than this, application was also made to a retrograde orbit having the same sidereal period as one of Darwin's orbits; and a comparison, illustrated by a diagram, was made of twelve positions in retrograde, direct, and undisturbed orbits having the same sidereal period. The fact

* *Proc. of the Fifth Internat. Congress of Math.*, vol. ii. p. 185.

that the deviations from uniform circular motion are much greater in direct orbits than they are in retrograde orbits having the same sidereal periods was shown in the clearest manner in this discussion.

In the *Transactions of the American Mathematical Society*, vol. xiii. (1912), pp. 96–108, the writer has treated a class of periodic orbits of superior planets, that is, periodic orbits which enclose both of the finite bodies. It was shown that there are three families (two of which are complex for small values of the parameter) in which the motion with respect to fixed axes is direct, and three in which it is retrograde. The orbits are all retrograde with respect to axes which rotate so as always to pass through the finite bodies. The first terms of the series for the co-ordinates were computed, and it was shown how they may be indefinitely continued.

3. The differential equations in rotating axes and canonical units for the motion of the infinitesimal body are

$$(1)\quad \begin{cases} \dfrac{d^2X}{dt^2} - 2\dfrac{dY}{dt} = \dfrac{\partial U}{\partial X}, \\ \dfrac{d^2Y}{dt^2} + 2\dfrac{dX}{dt} = \dfrac{\partial U}{\partial Y}, \\ U = \frac{1}{2}(X^2+Y^2) + \dfrac{1-\mu}{r_1} + \dfrac{\mu}{r_2}, \\ r_1^2 = (X+\mu)^2 + Y^2,\ r_2^2 = (X-1+\mu)^2 + Y^2. \end{cases}$$

These equations are equivalent to those treated by the writer in 1906 after the latter have been suitably specialised. The first terms of their periodic solution as given in the earlier paper, pp. 563–565, are

$$(2)\quad \begin{cases} X = \phi(t-t_0) = r\cos\left[\dfrac{t-t_0}{m} + w\right], \\ Y = \psi(t-t_0) = r\sin\left[\dfrac{t-t_0}{m} + w\right], \\ r = a\left\{1 - \left[\frac{1}{6} + \cos\dfrac{2}{m}(t-t_0)\right]\eta m^2 + \left[\frac{1}{3} + \frac{15}{16}M_1\cos\dfrac{1}{m}(t-t_0) - \frac{7}{6}\cos\dfrac{2}{m}(t-t_0)\right]\eta m^3 + \cdots\right\}, \\ w = \left\{\frac{11}{8}\sin\dfrac{2}{m}(t-t_0)\eta m^2 + \left[-\frac{15}{8}M_1\sin\dfrac{1}{m}(t-t_0) + \frac{13}{6}\sin\dfrac{2}{m}(t-t_0)\right]\eta m^3 + \cdots\right\}, \\ a = \left(\dfrac{m}{1+m}\right)\mu^{\frac{1}{3}}, \\ M_1 = \dfrac{a}{m^2}, \quad \eta = 1-\mu, \\ m = \text{the ratio of the synodic period of the satellite to the sidereal period of the finite bodies.} \end{cases}$$

The constants t_0 and m are arbitrary, with the single restriction that m must not be too large or the series will not converge. The equations giving a, M_1, and η express the constants of the earlier paper in terms of those used here. Since it is now supposed that the satellite is infinitesimal, M of the earlier paper must be replaced by zero. Only the first terms of the expressions for r and w are given here because of the complexity of those of higher order. It should also be noted that τ of the original paper is $(t-t_0)/m$ here. This reversion to the original independent variable, the time, is necessary for the later parts of this paper.

In order to reduce equations (2) to numbers it is only necessary to have the values of μ and m. When the sum of the masses of the Sun and Jupiter is taken as unity, the mass of Jupiter is $\mu = 1/1048{\cdot}35$. The mean daily motions of Jupiter and J VIII. are respectively

$$(3) \quad n' = +299''{\cdot}13, \quad n = -1748''{\cdot}19.$$

Therefore

$$(4) \quad m = \frac{n'}{n-n'} = -0{\cdot}146107, \; a = +0{\cdot}030340, \; \eta = 0{\cdot}99907.$$

Then, on using the more extended series for r and w given in the original paper, it is found that

$$(5) \begin{cases} r = +0{\cdot}030207 - 0{\cdot}000071\cos\frac{1}{m}(t-t_0) - 0{\cdot}000544\cos\frac{2}{m}(t-t_0) \\ \quad - 0{\cdot}000008\cos\frac{3}{m}(t-t_0) - 0{\cdot}000005\cos\frac{4}{m}(t-t_0) \\ \quad - 0{\cdot}000002\cos\frac{5}{m}(t-t_0) - 0{\cdot}000002\cos\frac{6}{m}(t-t_0) \ldots, \\ w = +0{\cdot}003933\sin\frac{1}{m}(t-t_0) + 0{\cdot}023325\sin\frac{2}{m}(t-t_0) \\ \quad + 0{\cdot}000355\sin\frac{3}{m}(t-t_0) + 0{\cdot}000240\sin\frac{4}{m}(t-t_0) \\ \quad + 0{\cdot}000010\sin\frac{5}{m}(t-t_0) + 0{\cdot}000003\sin\frac{6}{m}(t-t_0) \ldots. \end{cases}$$

The coefficients of the trigonometric functions in these series are power series in m. The coefficients of $\cos\frac{1}{m}(t-t_0)$, in the first equation, and of $\sin\frac{1}{m}(t-t_0)$, in the second, converge very slowly and are subject to some uncertainty. The coefficient of $\sin\frac{1}{m}(t-t_0)$ was transformed in two ways in the original paper so as to secure more rapid convergence. The transformed series are the ex-

pressions (53) and (54) of that paper. The coefficient in (5) is the mean of the two results given by those expressions.

The initial conditions as defined by (5) are

$$(6)\quad \begin{cases} r(0) = +0{\cdot}029575,\ r'(0) = 0,\ w(0) = 0,\ w'(0) = -0{\cdot}36053, \\ \mathrm{X}(0) = +0{\cdot}029575,\ \mathrm{X}'(0) = 0,\ \mathrm{Y}(0) = 0,\ \mathrm{Y}'(0) = -0{\cdot}213085. \end{cases}$$

The corresponding value of the Jacobian constant is $C = 3{\cdot}018917$. which is a little smaller than the values of C for the non-periodic orbit of J VIII. which Jackson found from the actual co-ordinates and components of velocity of the satellite at four dates.

In view of the fact that the expressions for the coefficients of $\sin\frac{1}{m}(t-t_0)$ and $\cos\frac{1}{m}(t-t_0)$ are slowly convergent, and that therefore the orbit may be somewhat in doubt, the orbit was computed* by mechanical quadratures from the initial conditions given in (6). The half period of the orbit sought is 0·4590. It was found that the satellite reached the x-axis at 0·4581, or a little too soon. At the time of crossing, the value of x' was +0·0007 instead of zero as it would have been in the exact periodic orbit. This shows that the adopted value of $w'(0)$ was a little too small, but the results are so nearly correct that it has not been thought worth while to make the slight alteration necessary to get the periodic orbit more accurately; the error would not be visible in a diagram. The computation shows that the limit of usefulness of the series (2) is reached for these values of m and μ.

It should be stated that in making the numerical calculations the procedure of Darwin has not been adopted. The rectangular co-ordinates were employed throughout without using the Jacobian integral to simplify the equations. This makes it possible to use the integral as a check, at any stage, on the accuracy of the work, and repeated tests show that the computed value of the Jacobian constant never differed from its initial value by more than one unit in the fifth significant figure. Under these circumstances the calculations may be regarded as fully trustworthy.

4. The stability of an orbit may be defined precisely if it is periodic. Deviations from the periodic solution (2) may be obtained by the substitution

$$(7)\quad \mathrm{X} = \phi(t-t_0) + x, \quad \mathrm{Y} = \psi(t-t_0) + y,$$

whence equations (1) become

$$(8)\quad \begin{cases} \dfrac{d^2x}{dt^2} - 2\dfrac{dy}{dt} = \dfrac{\partial^2 \mathrm{U}}{\partial \mathrm{X}^2}x + \dfrac{\partial^2 \mathrm{U}}{\partial \mathrm{X}\partial \mathrm{Y}}y + \ \cdot\ \cdot\ \cdot, \\ \dfrac{d^2y}{dt^2} + 2\dfrac{dx}{dt} = \dfrac{\partial^2 \mathrm{U}}{\partial \mathrm{X}\partial \mathrm{Y}}x + \dfrac{\partial^2 \mathrm{U}}{\partial \mathrm{Y}^2}y + \ \cdot\ \cdot\ \cdot\ \cdot \end{cases}$$

* I am greatly indebted to my assistant, Mr. W. L. Hart, who has made nearly all the computations contained in this paper.

It follows from the general theory of the nature of the dependence of the solutions of differential equations upon the initial values of the dependent variables that these equations can be integrated as power series in the initial values of x and y and their first derivatives. These series will converge for all values of t in a sufficiently restricted range. The smaller the initial values of x, y and their derivatives, the larger the range on t for which the series converge; and the initial values of the dependent variables can be taken so small that the solution will converge for any finite preassigned interval of time.

The terms of the solution of (8), which are of the first degree in x, y and their first derivatives, are defined by

$$(9)\quad \begin{cases} \dfrac{d^2x}{dt^2} - 2\dfrac{dy}{dt} = \dfrac{\partial^2 U}{\partial X^2}x + \dfrac{\partial^2 U}{\partial X \partial Y}y, \\ \dfrac{d^2y}{dt^2} + 2\dfrac{dx}{dt} = \dfrac{\partial^2 U}{\partial X \partial Y}x + \dfrac{\partial^2 U}{\partial Y^2}y. \end{cases}$$

In the second derivatives of U the quantities X and Y are replaced by ϕ and ψ respectively. Hence these differential equations are linear, homogeneous, and have periodic coefficients. The solutions of equations (9) in general have the form

$$(10)\quad \begin{cases} x = A_1 e^{\beta_1(t-t_0)}\xi_1(t-t_0) + A_2 e^{\beta_2(t-t_0)}\xi_2(t-t_0) \\ \qquad + A_3 e^{\beta_3(t-t_0)}\xi_3(t-t_0) + A_4 e^{\beta_4(t-t_0)}\xi_4(t-t_0), \\ y = A_1 e^{\beta_1(t-t_0)}\eta_1(t-t_0) + A_2 e^{\beta_2(t-t_0)}\eta_2(t-t_0) \\ \qquad + A_3 e^{\beta_3(t-t_0)}\eta_3(t-t_0) + A_4 e^{\beta_4(t-t_0)}\eta_4(t-t_0), \end{cases}$$

where $A_1, \ldots, A_4$ are arbitrary constants, $\beta_1, \ldots, \beta_4$ are constants depending on the differential equations, and $\xi_1, \ldots, \xi_4$, $\eta_1, \ldots, \eta_4$ are periodic functions of $t - t_0$ having the period of ϕ and ψ. The form of the solution may be different if two or more of the β_i are equal. The solution (2) is said to be stable if the β_i are zero or pure imaginaries, for then the exponentials have the modulus unity, and x, y and their derivatives do not depart far from their initial values, at least so far as their values are determined by (9). If $A_1, \ldots, A_4$ are small, the higher terms of the solution of (8) are also small, at least provided t is not too remote from its initial value; hence there is a real meaning to this definition of stability, which is now classic, and which is written down here explicitly only in order to make precise the notation in what follows.

The orbit of J VIII. whose stability is in question is not periodic, and hence the criterion of stability does not strictly apply. But the characteristics of its motion will be analogous to those of a periodic orbit in the same general neighbourhood. This is intuitionally clear, and it also follows from the fundamental theorem that the solutions of the differential equations of the type of (1) are regular analytic functions of the initial values of the

dependent variables. Hence it is reasonable to conclude that if the periodic orbit is decidedly stable, then a neighbouring non-periodic orbit is also stable. The neighbouring non-periodic orbit may be obtained rigorously by integrating equations (8) with suitable initial conditions. The terms of lowest degree in this solution are precisely (10). If the β_i are purely imaginary, the higher terms are numerically less important than they are if this condition is not satisfied. If the A_i are small, that is, if the non-periodic orbit does not differ much from the periodic orbit, the higher terms are much less important numerically than they are if the A_i are not all small. Hence the criterion is the more certain the nearer the non-periodic orbit is to the periodic orbit.

5. As was shown by Hill in his celebrated memoir on the Lunar Theory, in *Acta Mathematica*, vol. viii., the solutions of (9) which do not have the period of the periodic solution (2) can be made to depend upon the solution of

$$(11) \quad \frac{d^2u}{dt^2} + [A_0 + \theta_1(t-t_0)m + \theta_2(t-t_0)m^2 + \ \ldots\ ]u = 0,$$

where $\theta_1(t-t_0)$, $\theta_2(t-t_0)$, . . . are periodic with the period of ϕ and ψ, or $2\pi m$ in the present problem. The coefficient of u in (11) is a very complicated function of the coefficients of (9) and can be formed only by considerable labour. Hill gave a method of solving this equation depending upon the solution of an infinite determinant. In order to apply this method the coefficient of u was required as a Fourier series rather than as a power series in m. The method of Hill was followed by Darwin in determining the stability of his periodic orbits.

When the coefficient of u is expansible as a converging power series in a parameter m, as in the Lunar Theory, the solution may be obtained conveniently by a method given by the writer in a paper on the spherical pendulum in *Rendiconti del Circolo Matematico di Palermo*, vol. xxxii. (1911). But if the period of the periodic orbit is long, the coefficient of u may not be expansible as a power series in m, and its representation by Fourier series may converge very slowly. Darwin found many such cases, and either determined the stability only approximately after much labour, or left it undetermined. The problem is to avoid these difficulties.

The solution of equation (11) may be written in the form

$$u = C_1 e^{\beta_1(t-t_0)} u_1(t-t_0) + C_2 e^{\beta_2(t-t_0)} u_2(t-t_0),$$

where C_1 and C_2 are arbitrary constants, β_1 and β_2 are constants depending on the coefficients of (11), and $u_1(t-t_0)$ and $u_2(t-t_0)$ are periodic functions with the period $2\pi m$. It was shown in the paper on the spherical pendulum that β_1 and β_2 are determined by [eq. (42)]

$$(12) \quad \cosh 2\beta\pi m = u_0(2\pi m),$$

where u_0 is the solution of (11) determined by the initial conditions $u_0(0) = 1$, $u'_0(0) = 0$. Therefore $\beta_1 = -\beta_2$.

If the periodic orbit whose stability is in question has been found by numerical experiment, the coefficient of u in (11) will be known for a series of values of $t - t_0$ corresponding to those for which the co-ordinates in the periodic orbit were computed. If the periodic orbit is given by the series (2), the coefficient of u in (11) can be obtained for any desired values of $t - t_0$. In either case there is no trouble whatever in computing $u_0(2\pi m)$ by mechanical quadratures and determining β by (12).

6. The question arises whether the complicated transformations may not be avoided which are necessary in going from equations (9) to equations (11). The answer is in the affirmative. As preliminary, construct four solutions of (9) defined by the initial conditions

$$(13)\quad \begin{cases} \text{1st sol.} & x_1(0)=1, & x'_1(0)=0, & y_1(0)=0, & y'_1(0)=0\,; \\ \text{2nd sol.} & x_2(0)=0, & x'_2(0)=1, & y_2(0)=0, & y'_2(0)=0\,; \\ \text{3rd sol.} & x_3(0)=0, & x'_3(0)=0, & y_3(0)=1, & y'_3(0)=0\,; \\ \text{4th sol.} & x_4(0)=0, & x'_4(0)=0, & y_4(0)=0, & y'_4(0)=1. \end{cases}$$

These solutions form a fundamental set whose determinant is unity. It was proved in a paper on linear differential equations with periodic coefficients * by MacMillan and the writer that in equations of the type of (9) the determinant of a fundamental set of solutions is constant. All the solutions of (9) can be expressed linearly and homogeneously in terms of the solutions defined by (13).

In order to bring into evidence the exponents β_i of equations (10), make the transformation

$$(14)\quad x = e^{\beta(t-t_0)}\xi, \qquad y = e^{\beta(t-t_0)}\eta,$$

where β is an undetermined constant. Equations (9) become, as a consequence of this substitution,

$$(15)\quad \begin{cases} \dfrac{d^2\xi}{dt^2} + 2\beta\dfrac{d\xi}{dt} - 2\dfrac{d\eta}{dt} = \left[-\beta^2 + \dfrac{\partial^2 U}{\partial X^2}\right]\xi + \left[2\beta + \dfrac{\partial^2 U}{\partial X \partial Y}\right]\eta, \\ \dfrac{d^2\eta}{dt^2} + 2\beta\dfrac{d\eta}{dt} + 2\dfrac{d\xi}{dt} = \left[-2\beta + \dfrac{\partial^2 U}{\partial X \partial Y}\right]\xi + \left[-\beta^2 + \dfrac{\partial^2 U}{\partial Y^2}\right]\eta. \end{cases}$$

It follows from the relations (14) that any solution of (15) can be expressed as $e^{-\beta(t-t_0)}$ times a linear function of the solutions defined by the initial conditions (13). Therefore the general solution of (15) has the form

$$(16)\quad \begin{cases} \xi = e^{-\beta(t-t_0)}[a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4], \\ \xi' = e^{-\beta(t-t_0)}[a_1x'_1 + a_2x' + a_3x'_3 + a_4x'_4] \\ \qquad - \beta e^{-\beta(t-t_0)}[a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4], \\ \eta = e^{-\beta(t-t_0)}[a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4], \\ \eta' = e^{-\beta(t-t_0)}[a_1y'_1 + a_2y'_2 + a_3y'_3 + a_4y'_4] \\ \qquad - \beta e^{-\beta(t-t_0)}[a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4] \end{cases}$$

* *The American Journal of Mathematics*, vol. xxxiii. (1911), pp. 63-96.

Since the coefficients in (15) are periodic with the period $2\pi m$, necessary and sufficient conditions that ξ and η shall be periodic with the period $2\pi m$ are

$$(17)\quad \begin{cases} \xi(2\pi m) - \xi(0) = 0, & \xi'(2\pi m) - \xi'(0) = 0, \\ \eta(2\pi m) - \eta(0) = 0, & \eta'(2\pi m) - \eta'(0) = 0. \end{cases}$$

These conditions become as a consequence of (16) and the initial conditions (13)

$$(18)\quad \begin{cases} [x_1(2\pi m) - \omega]a_1 + x_2(2\pi m)a_2 + x_3(2\pi m)a_3 + x_4(2\pi m)a_4 = 0, \\ x'_1(2\pi m)a_1 + [x'_2(2\pi m) - \omega]a_2 + x'_3(2\pi m)a_3 + x'_4(2\pi m)a_4 = 0, \\ y_1(2\pi m)a_1 + y_2(2\pi m)a_2 + [y_3(2\pi m) - \omega]a_3 + y_4(2\pi m)a_4 = 0, \\ y'_1(2\pi m)a_1 + y'_2(2\pi m)a_2 + y'_3(2\pi m)a_3 + [y'_4(2\pi m) - \omega]a_4 = 0, \end{cases}$$

where

$$(19)\quad \omega = e^{2\pi\beta m}.$$

In order that equations (18) may have any solution other than $a_1 = a_2 = a_3 = a_4 = 0$, the determinant of their coefficients must be zero, or

$$(20)\quad D(\omega) = \begin{vmatrix} x_1(2\pi m) - \omega, & x_2(2\pi m), & x_3(2\pi m), & x_4(2\pi m) \\ x'_1(2\pi m), & x'_2(2\pi m) - \omega, & x'_3(2\pi m), & x'_4(2\pi m) \\ y_1(2\pi m), & y_2(2\pi m), & y_3(2\pi m) - \omega, & y_4(2\pi m) \\ y'_1(2\pi m), & y'_2(2\pi m), & y'_3(2\pi m), & y'_4(2\pi m) - \omega \end{vmatrix} = 0$$

The coefficient of ω^4 in the expansion of $D(\omega)$ is unity; and since the determinant of the fundamental set is constant, the term independent of ω is also unity. It will be shown in the next section that this determinant has two roots which are unity, and that the other two are reciprocals of each other. These properties serve as very good checks on the computations which can be made with the initial conditions (13), by mechanical quadratures, without any difficulty. Hence equations (20) and (19) may be used for determining β instead of making transformations to (11) and then using (12).

7. The periodic solution (2) contains the two arbitrary constants t_0 and m. The functions ϕ and ψ are analytic in t_0 and m and are regular at any finite $t_0 = t_0^{(0)}$, $m = m_0 \pm 0$. Consequently, ϕ and ψ are expansible as power series in $t_0 - t_0^{(0)}$ and $m - m_0$ of the form

$$(21)\quad \begin{cases} \phi = \phi(t - t_0^{(0)}, m_0) + \dfrac{\partial\phi}{\partial t_0}(t_0 - t_0^{(0)}) + \dfrac{\partial\phi}{\partial m}(m - m_0) + \ \ldots, \\ \psi = \psi(t - t_0^{(0)}, m_0) + \dfrac{\partial\psi}{\partial t_0}(t_0 - t_0^{(0)}) + \dfrac{\partial\psi}{\partial m}(m - m_0) + \ \ldots. \end{cases}$$

The linear and higher terms of these series are the deviations from the periodic solution which exists for $t_0 = t_0^{(0)}$, $m = m_0$. Therefore they are periodic solutions of equations (8), and their linear terms are solutions of (9).

It follows that two solutions of equations (9) are

$$(22)\quad \begin{cases} x_1 = \dfrac{\partial \phi}{\partial t_0} = -\dfrac{\partial \phi}{\partial t}, & y_1 = \dfrac{\partial \psi}{\partial t_0} = -\dfrac{\partial \psi}{\partial t}, \\ x_2 = \dfrac{\partial \phi}{\partial m} = \phi_1 + (t - t_0)\phi_2, & y_2 = \psi_1 + (t - t_0)\psi_2, \end{cases}$$

where ϕ_1, ϕ_2, ψ_1, and ψ_2 are periodic with the period $2\pi m$. The ϕ_1 and ψ_1 are obtained by taking the derivative of ϕ and ψ so far as they involve m not under the trigonometric functions; and ϕ_2 and ψ_2 are obtained by taking the derivative with respect to m so far as it occurs under the trigonometric signs. It follows from the properties of ϕ and ψ and the method of obtaining ϕ_1, ϕ_2, ψ_1, and ψ_2 that ϕ_1 and ψ_2 are even functions of $t - t_0$, while ϕ_2 and ψ_1 are odd functions of $t - t_0$. Moreover,

$$(23)\quad x_1 = -\frac{\partial \phi}{\partial t} = m\phi_2, \qquad y_1 = -\frac{\partial \psi}{\partial t} = m\psi_2.$$

Therefore

$$(24)\quad \begin{cases} x_1(0) = 0,\ x'_1(0) = m\phi'_2(0),\ y_1(0) = m\psi_2(0),\ y'_1(0) = 0, \\ x_2(0) = \phi_1(0),\ x'_2(0) = 0,\ y_2(0) = 0,\ y'_2(0) = \psi_2(0). \end{cases}$$

The two solutions (22) together with those defined by the first and second, first and third, second and fourth, and third and fourth sets of initial conditions (13) form fundamental sets of solutions because their determinants at $t = t_0$ are respectively

$$m\psi_2(0)[\psi'_1(0) + \psi_2(0)],\ -m\phi'_2(0)[\psi'_1(0) + \psi_2(0)],$$
$$m\psi_2(0)\phi_1(0),\ -m\phi'_2(0)\phi_1(0),$$

all of which are distinct from zero. On making use of the fact that ϕ_1, ϕ_2, ψ_1, and ψ_2 are periodic, referring to the initial conditions (24), and taking the solutions defined by the second and fourth conditions of (13), the expression for $D(\omega)$, equation (20), becomes, by reduction similar to that used in obtaining (20) from (17),

$$(25)\quad D(\omega) = m(1-\omega)^2 \begin{vmatrix} 0, & \phi_1(0), & x_2(2\pi m), & x_4(2\pi m) \\ \phi'_2(0), & 0, & x'_2(2\pi m) - \omega, & x'_4(2\pi m) \\ \psi_2(0), & 0, & y_2(2\pi m), & y_4(2\pi m) \\ 0, & \psi'_1(0) + \psi_2(0), & y'_2(2\pi m), & y'_4(2\pi m) - \omega \end{vmatrix} = 0.$$

The two roots unity are now in evidence.

Since the determinant of a fundamental set of solutions of linear differential equations of the type of (9) is constant, it follows from (13) that in $D(\omega)/m(1-\omega)^2$ the coefficient of ω^2 is

identical with the absolute term. Hence $D(\omega)$ takes the simple form

$$(26)\quad D(\omega) = m\phi_1(0)\psi_2(0)\,(1-\omega^2)\,[\omega^2 + 2A\omega + 1] = 0,$$

where

$$(27)\quad 2A = \frac{1}{\psi_2(0)}\begin{vmatrix}\phi'_2(0), & x'_2(2\pi m)\\ \psi_2(0), & y_2(2\pi m)\end{vmatrix} - \frac{1}{\phi_1(0)}\begin{vmatrix}\phi_1(0), & x_4(2\pi m)\\ \psi'_1(0)+\psi_2(0), & y'_4(2\pi m)\end{vmatrix}.$$

The condition that the constant term of the quadratic factor of (26) shall be unity is

$$\frac{1}{\phi_1(0)\psi_2(0)}\begin{vmatrix} 0, & \phi_1(0), & x_2(2\pi m), & x_4(2\pi m)\\ \phi'_2(0), & 0, & x'_2(2\pi m), & x'_4(2\pi m)\\ \psi_2(0), & 0, & y_2(2\pi m), & y_4(2\pi m)\\ 0, & \psi'_1(0)+\psi_2(0), & y'_2(2\pi m), & y'_4(2\pi m)\end{vmatrix} = 1,$$

which can serve as a check on the computation of the quantities appearing in this determinant.

It follows from (2) and (23) that

$$(28)\quad \begin{cases} m\phi'_2(0) = -\left(\dfrac{\partial^2\phi}{\partial t^2}\right)_0 = -\left(\dfrac{\partial^2 r}{\partial t^2}\right)_0 + r(0)\left[\dfrac{1}{m} + w'(0)\right]^2,\\ m\psi_2(0) = -\left(\dfrac{\partial\psi}{\partial t}\right)_0 = -r(0)\left[\dfrac{1}{m} + w'(0)\right].\end{cases}$$

Let the derivative of a function with respect to m, except in so far as m appears under the trigonometric functions, be designated by the usual symbols enclosed in parentheses. Then it follows from (23) and the notations which are explained following these equations that

$$(29)\quad \begin{cases}\phi_1(0) = \left(\dfrac{\partial r}{\partial m}\right)_0,\\ \psi'_1(0) = \left[\dfrac{d}{dt}\left(\dfrac{\partial\psi}{\partial m}\right)\right]_0 = \left(\dfrac{\partial r}{\partial m}\right)_0\left[\dfrac{1}{m} + w'(0)\right] + r(0)\left[\dfrac{d}{dt}\left(\dfrac{\partial w}{\partial m}\right)\right]_0.\end{cases}$$

It is found from the series in the *Transactions*, of which the first terms are given in (2), for the data (3) and (4), that

$$\phi_1(0) = -0{\cdot}148,\quad \psi'_1(0) = +1{\cdot}195,$$
$$m\phi'_2(0) = +1{\cdot}424,\quad m\psi_2(0) = +0{\cdot}213.$$

With the second and fourth sets of initial conditions (13), which give smallest numbers in practice, it is found by mechanical quadratures that

$$x_2(2\pi m) = -{\cdot}075,\quad x'_2(2\pi m) = +\ 1{\cdot}729,\quad y_2(2\pi m) = +0{\cdot}165,\quad y'_2(2\pi m) = -0$$
$$x_4(2\pi m) = -{\cdot}178,\quad x'_4(2\pi m) = -24{\cdot}201,\quad y_4(2\pi m) = -3{\cdot}970,\quad y'_4(2\pi m) = +0$$

Then the determinant (25) becomes

$$\begin{vmatrix} 0, & -0\cdot148, & -0\cdot075, & -0\cdot178 \\ +1\cdot424, & 0, & +1\cdot729-\omega, & -24\cdot201 \\ +0\cdot213, & 0, & +0\cdot165, & -3\cdot970 \\ 0, & -0\cdot263, & -0\cdot378, & +0\cdot235-\omega \end{vmatrix} = 0.$$

The coefficient of ω^2 is $-0\cdot213\times0\cdot148=-0\cdot0315$. The term independent of ω is $-0\cdot0289$. The small difference of 0·0026 between these numbers is due to the accumulated errors which arise in the third and fourth decimals. They appear especially when the numbers have one or two digits to the left of the decimal point. These slight errors are of no importance in determining the character of the stability, and besides the results are given two independent checks in the next section of this paper.

The coefficient of ω is 0·0368. Hence the equation for ω becomes (on giving the term independent of ω the same value as the coefficient of ω^2)

$$\omega^2+2A\omega+1\equiv\omega^2-1\cdot168\omega+1=0.$$

It follows from (19) that this equation can be written in the form

$$\cosh 2\pi\beta m=-A=+0\cdot584.$$

Let $\beta=\alpha\sqrt{-1}$. Then

$$(30)\quad \cos\pi\alpha m=+0\cdot584,\quad 2\pi\alpha m=54^\circ\ 16'.$$

When the satellite moves in an undisturbed orbit all the roots of the determinant in ω are unity, and the corresponding value of $-A$ is unity. The difference between unity and 0·584 measures the approach toward instability which begins, for A continuously diminishing, when $-A$ passes -1. The orbit would be unstable also if $-A$ were greater than unity. The first type is equivalent to that which Sir George Darwin * called "uneven instability," and the second type is that which he called "even instability." The change in A is toward "uneven instability," but the orbit is very stable because the change in A from its value in undisturbed motion is only about one-fifth of that required before instability sets in.

8. The only difficulty in determining ω from (20) or (25) is in computing the co-ordinates in the particular solutions at $t-t_0=2\pi m$. It will now be shown how the computation may be made to depend upon the solutions at the mid-period $t-t_0=\pi m$. This reduces the labour of computation by more than one-half, because the whole interval covered is only half as great, and the computation of the first part is easier than that of the second part. The reason is that the solutions are partly of the nature of the products of $t-t_0$ and periodic functions, as is shown by the second equations of (22), and the longer they are continued the larger become the variations in the co-ordinates.

* *Acta Mathematica*, vol. xxi. pp. 147–8.

It follows from the definition of U given in (1) and the properties of the periodic solution given in (2), that for the periodic orbit the functions $\partial^2 U/\partial X^2$ and $\partial^2 U/\partial Y^2$ are even functions of $t - t_0$, and that $\partial^2 U/\partial X \partial Y$ is an odd function of $t - t_0$. On making use of these facts it is easy to prove that in the solutions of (9) which are defined by the first and fourth sets of initial conditions (13), x is an even function of $t - t_0$ and y is an odd function of $t - t_0$, and that in the two other solutions defined by (13), x is an odd function of $t - t_0$ and y is an even function of $t - t_0$.

Obviously, therefore, instead of equations (17), sufficient conditions that ξ and η shall be periodic with the period $2\pi m$ are

$$\begin{cases} \xi(\pi m) - \xi(-\pi m) = 0, & \xi'(\pi m) - \xi'(-\pi m) = 0, \\ \eta(\pi m) - \eta(-\pi m) = 0, & \eta'(\pi m) - \eta'(-\pi m) = 0. \end{cases}$$

These conditions become as a consequence of (16)

$$(31)\quad \begin{cases} [x_1(\pi m) - \omega x_1(-\pi m)]a_1 + [x_2(\pi m) - \omega x_2(-\pi m)]a_2 \\ \quad + [x_3(\pi m) - \omega x_3(-\pi m)]a_3 + [x_4(\pi m) - \omega x_4(-\pi m)]a_4 = 0, \\ [x'_1(\pi m) - \omega x'_1(-\pi m)]a_1 + [x'_2(\pi m) + \omega x'_2(-\pi m)]a_2 \\ \quad + [x'_3(\pi m) - \omega x'_3(-\pi m)]a_3 + [x'_4(\pi m) - \omega x'_4(-\pi m)]a_4 = 0, \\ [y_1(\pi m) - \omega y_1(-\pi m)]a_1 + [y_2(\pi m) - \omega y_2(-\pi m)]a_2 \\ \quad + [y_3(\pi m) - \omega y_3(-\pi m)]a_3 + [y_4(\pi m) - \omega y_4(\pi m)]a_4 = 0, \\ [y'_1(\pi m) - \omega y'_1(-\pi m)]a_1 + [y'_2(\pi m) - \omega y'_2(-\pi m)]a_2 \\ \quad + [y'_3(\pi m) - \omega y'_3(-\pi m)]a_3 + [y'_4(\pi m) - \omega y'_4(-\pi m)]a_4 = 0. \end{cases}$$

Since x_1, x_4, y_2, and y_3 are even functions of $t - t_0$, it follows that their values at $t - t_0 = \pi m$ and at $t - t_0 = -\pi m$ are equal; and since x_2, x_3, y_1, and y_4 are odd functions, their values at $t - t_0 = \pi m$ and at $t - t_0 = -m\pi$ are equal numerically but opposite in sign. Hence the determinant of the coefficients of $a_1, \ldots, a_4$ in (31) becomes

$$(32)\quad D(\omega) = \begin{vmatrix} (1-\omega)x_1(\pi m), & (1+\omega)x_2(\pi m), & (1+\omega)x_3(\pi m), & (1-\omega)x_4(\pi m) \\ (1+\omega)x'_1(\pi m), & (1-\omega)x'_2(\pi m), & (1-\omega)x'_3(\pi m), & (1+\omega)x'_4(\pi m) \\ (1+\omega)y_1(\pi m), & (1-\omega)y_2(\pi m), & (1-\omega)y_3(\pi m), & (1+\omega)y_4(\pi m) \\ (1-\omega)y'_1(\pi m), & (1+\omega)y'_2(\pi m), & (1+\omega)y'_3(\pi m), & (1-\omega)y'_4(\pi m) \end{vmatrix}.$$

It follows easily from the form of $D(\omega)$ that the coefficients of ω^4 and the constant term are identical. Likewise the coefficients of ω^3 and ω are identical. Therefore $D(\omega) = 0$ is a reciprocal equation. The computation of the elements of the determinant $D(\omega)$ is checked by the fact that it must have two roots unity.

If the solutions (22) are used in place of the first and third of those defined by the initial conditions (13), the equation $D(\omega) = 0$ becomes simply

$$D(\omega) = m(1-\omega)^2 \begin{vmatrix} 0, & \phi_1(\pi m), & (1+\omega)x_2(\pi m), & (1+\omega)x_4(\pi m) \\ \phi'_2(\pi m), & 0, & (1+\omega)x'_2(\pi m), & (1+\omega)x'_4(\pi m) \\ {}_2(\pi m), & 0, & (1-\omega)y_2(\pi m), & (1+\omega)y_4(\pi m) \\ 0, & \psi'_1(\pi m)+\psi_2(\pi m), & (1+\omega)y'_2(\pi m), & (1-\omega)y'_4(\pi m) \end{vmatrix} = 0,$$

in which the two roots unity are in evidence. Since the coefficient of ω^2 and the constant term in this equation are equal, as follows from the elementary properties of determinants, $D(\omega) = 0$ is a reciprocal equation.

The method of this section was also applied to the determination of the stability of J VIII. It was found from the more extended form of the series (2) given in the original paper that

$$(34) \quad \begin{cases} \phi_1(\pi m) = +0{\cdot}1511, & \psi'_1(\pi m) = -1{\cdot}1930, \\ m\phi'_2(\pi m) = -1{\cdot}4200, & m\psi_2(\pi m) = -0{\cdot}2127. \end{cases}$$

Three solutions of equations (9), defined by the second, third, and fourth sets of initial conditions of (13), were computed. They were respectively, at $t = \pi m$,

$x_2(\pi m) = -0{\cdot}0505$, $x'_2(\pi m) = -3{\cdot}4260$, $y_2(\pi m) = -0{\cdot}6431$, $y'_2(\pi m) = -0{\cdot}2330$;
$x_3(\pi m) = +0{\cdot}3461$, $x'_3(\pi m) = +16{\cdot}3270$, $y_3(\pi m) = +3{\cdot}3155$, $y'_3(\pi m) = +1{\cdot}5860$;
$x_4(\pi m) = +0{\cdot}6501$, $x'_4(\pi m) = +8{\cdot}5080$, $y_4(\pi m) = +1{\cdot}0632$, $y'_4(\pi m) = +2{\cdot}0430$.

On using (34) and the first and third of these solutions, the determinant (33) becomes

$$(36) \quad \begin{vmatrix} 0, & 0{\cdot}1511, & 0{\cdot}0505(1+\omega), & 0{\cdot}6501(1-\omega) \\ -1{\cdot}4200, & 0, & 3{\cdot}4260(1-\omega), & 8{\cdot}5080(1+\omega) \\ -0{\cdot}2127, & 0, & 0{\cdot}6431(1-\omega), & 1{\cdot}0632(1+\omega) \\ 0, & 0{\cdot}2623, & 0{\cdot}2330(1+\omega), & 2{\cdot}0430(1-\omega) \end{vmatrix} = 0.$$

The coefficient of ω^2 in this determinant is 0·0316, and the term independent of ω is 0·0312. The agreement is very satisfactory. The coefficient of ω is, in the more general notation of (33), after some elementary reductions

$$2A = 2 \begin{vmatrix} 0, & \phi_1(\pi m), & +x_2(\pi m), & x_4(\pi m) \\ \phi'_2(\pi m), & 0, & -x'_2(\pi m), & x'_4(\pi m) \\ \psi_2(\pi m), & 0, & -y_2(\pi m), & y_4(\pi m) \\ 0, & \psi'_1(\pi m)+\psi_2(\pi m), & +y'_2(\pi m), & y'_4(\pi m) \end{vmatrix}$$

Equation (36) becomes, on reducing the coefficient of ω^2 to unity,

$$\omega^2 + 2A\omega + 1 = \omega^2 - 1{\cdot}190\omega + 1 = 0;$$

whence

$$(37) \quad \cos 2\pi\alpha m = +0{\cdot}595, \quad 2\pi\alpha m = 53^\circ\ 29',$$

which agrees closely with the results in (30).

On using (34) and the second and third solutions of (35), it is found similarly that

$$\begin{vmatrix} 0, & 0{\cdot}1511, & 0{\cdot}3461(1+\omega), & 0{\cdot}6501(1-\omega) \\ -1{\cdot}4200, & 0, & 16{\cdot}3270(1-\omega), & 8{\cdot}5080(1+\omega) \\ -0{\cdot}2127, & 0, & 3{\cdot}3155(1-\omega), & 1{\cdot}0632(1+\omega) \\ 0, & 0{\cdot}2613, & 1{\cdot}5860(1+\omega), & 2{\cdot}0430(1-\omega) \end{vmatrix} = 0.$$

The reduction of this determinant gives

$$(38)\quad \begin{cases} \omega^2 + 2A\omega + 1 = \omega^2 - 1{\cdot}172\omega + 1 = 0, \\ \cos 2\pi am = +0{\cdot}586, \quad 2\pi am = 54^\circ\ 8'. \end{cases}$$

The agreement of these results with those in (30) and (37) is very satisfactory, and they prove that the orbit of J VIII. is very stable.

9. After it was found that the orbit of J VIII. is very stable it was decided to examine the stability of a direct orbit having the same sidereal period. The first problem was to find an orbit which is periodic and has the required period. The series (2) in the more extended form of the original paper were first tried, and it was found that they do not converge, at least with sufficient rapidity to be useful. Hence it was necessary to make numerical experiments.

There is ordinarily no difficulty in discovering a periodic orbit by numerical experiments; but when the period is prescribed, as in the present case, a large amount of labour is involved. Another complicating factor, which of course was not foreseen, is that the "periodic orbit A" (Darwin's terminology) had departed for the required period so far from the circular form that it entirely escaped discovery; and orbits B and C were so much alike that successive trials gave little indication as to how the initial conditions should be corrected. Mr. Hart found it necessary to make eleven trials before a satisfactory result was reached.

The final nearly periodic solution was defined by the initial conditions

$$(39)\quad X(0) = 0{\cdot}029240, \quad X'(0) = 0, \quad Y(0) = 0, \quad Y'(0) = 0{\cdot}15800,$$

and the corresponding value of the Jacobian constant was

$$C = 3{\cdot}0390.$$

The final computed value of C was found to be 3·0391, which shows the high degree of accuracy of the numerical work.

The half-synodic period of a direct satellite having the same sidereal period as J VIII. is found by the data of (3) to be

$$\frac{P}{2} = 0{\cdot}64851.$$

With the initial conditions (39) the infinitesimal body crossed the x-axis at $t - t_0 = 0{\cdot}64361$ at an angle of 90° 11′. The period is a little too short, which favours stability, and the orbit is not strictly

periodic. A slightly greater initial value of Y′ would have given a more nearly periodic orbit and a little longer half period, but the required changes are so small it was not thought worth while to make them.

The following table gives the values of the co-ordinates and the derivatives for the periodic orbit at a considerable number of epochs :—

$t-t_0$.	X.	Y.	X′.	Y′.
·000	·029240	·000000	·00000	·15800
·025	·029018	·003938	− ·01765	·15656
·050	·028361	·007805	− ·03481	·15232
·075	·027287	·011532	− ·05105	·14547
·100	·025821	·015059	− ·06602	·13634
·125	·023999	·018332	− ·07950	·12526
·150	·021859	·021307	− ·09136	·11261
·175	·019444	·023950	− ·10157	·09872
·200	·016794	·026235	− ·11017	·08388
·225	·013948	·028138	− ·11721	·06833
·250	·010945	·029647	− ·12279	·05227
·275	·007820	·030749	− ·12696	·03580
·300	·004607	·031435	− ·12979	·01903
·325	·001339	·031699	− ·13129	·00203
·350	− ·001948	·031536	− ·13145	− ·01515
·375	− ·005222	·030942	− ·13019	− ·03247
·400	− ·008445	·029912	− ·12743	− ·04983
·425	− ·011580	·028450	− ·12302	− ·06714
·450	− ·014582	·026557	− ·11679	− ·08422
·475	− ·017403	·024242	− ·10857	− ·10085
·500	− ·019992	·021521	− ·09820	− ·11668
·525	− ·022294	·018418	− ·08557	− ·13130
·550	− ·024251	·014971	− ·07068	− ·14420
·575	− ·025809	·011227	− ·05368	− ·15482
·600	− ·026919	·007253	− ·03489	− ·16263
·625	− ·027543	·003124	− ·01483	− ·16714
·64375	− ·027676	− ·000023	+ ·00065	− ·16816

With the initial conditions (13) the solutions of (9) at

$t - t_0 = \pi m$, for the deviations from the orbit given in the table, which were computed by mechanical quadratures, are respectively

$$\begin{cases} x_1(\pi m) = -2{\cdot}3305, & x'_1(\pi m) = -73{\cdot}186, & y_1(\pi m) = 14{\cdot}9280, & y'_1(\pi m) = +9{\cdot}79 \\ x_2(\pi m) = +0{\cdot}2113, & x'_2(\pi m) = -1{\cdot}710, & y_2(\pi m) = 0{\cdot}5548, & y'_2(\pi m) = -1{\cdot}46 \\ x_3(\pi m) = +0{\cdot}9459, & x'_3(\pi m) = -2{\cdot}5345, & y_3(\pi m) = 1{\cdot}4463, & y'_3(\pi m) = -6{\cdot}56 \\ x_4(\pi m) = -0{\cdot}5154, & x'_4(\pi m) = -11{\cdot}191, & y_4(\pi m) = 2{\cdot}2762, & y'_4(\pi m) = +2{\cdot}59 \end{cases}$$

With these data the determinant (32) becomes

$$(40) \quad D(\omega) = \omega^4 - 4{\cdot}256\omega^3 + 6{\cdot}584\omega^2 - 4{\cdot}256\omega + 1 = 0.$$

The checks on this equation are that the coefficient of ω^4 must be unity and it must contain $(\omega - 1)^2$ as a factor, at least for the periodic orbit. It was found that the coefficient of ω^4 differs from unity by less than four units in the fourth significant figure. Since some of the numbers which enter the determinant are large, this result is satisfactory. This check is on the computation and does not depend upon the fact that the orbit under examination is periodic. Furthermore it is found that $D(1) = 0{\cdot}072$, $D'(1) = 0{\cdot}144$. Hence $(\omega - 1)^2$ is not exactly a factor of $D(\omega)$, but it was not expected that it would be, because the orbit under examination was not strictly periodic. Nevertheless, the condition is so nearly satisfied that the remaining roots of (40) will be given with sufficient accuracy to determine the nature of the stability of the periodic orbit.

After dividing out $(\omega - 1)^2$ from $D(\omega) = 0$, the resulting quadratic equation is

$$\omega^2 - 2{\cdot}26\omega + 1{\cdot}06 = 0,$$

whence

$$(41) \quad -A = +1{\cdot}13.$$

The orbit is therefore unstable with even instability. Sir George Darwin found * that the orbits of "Satellites B" are unstable with even instability. The periodic orbit which has been discovered and examined therefore belongs to the family B. Since an orbit of the family B can appear only if one of family C also appears, it follows that there is still another periodic orbit. And it follows from Poincaré's theorem † on the exchange of stabilities when two series of orbits pass through a common form that orbit C is stable. Since $-A$, equation (41), is not much greater than unity, the value of $-A$ belonging to the orbit C will be not much smaller than unity, and the orbit will be stable. However, since it will be near the limiting value unity at which stability ceases, the stability will not be great.

* *Acta Mathematica*, vol. xxi. pp. 211–217.

† *Les Methodes Nouvelles de la Mécanique Céleste*, vol. iii. § 378.

10. As a summary, it may be stated it has been shown in this paper how to determine by new methods whether a periodic orbit is stable or unstable. An application to a retrograde periodic orbit having the period of J VIII. shows that this orbit is very stable. A direct periodic orbit having approximately the period of J VIII. was computed and it was found to be unstable with even instability. It was inferred from Sir George Darwin's computations that this orbit belongs to his family B, and then from the general theory and Poincaré's theorem on the exchanges of stabilities that there exists an orbit C having the period of J VIII. which is stable. But none of the known satellites belongs to the type of family C.

A Proposal for the Comparison of the Stellar Magnitude Scales of the different Observatories taking part in the Astrographic Catalogue. Fourth Note: The Cape Magnitudes. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The previous notes are in *Monthly Notices*, lxix. p. 392, lxxii. p. 464, and lxxii. p. 700. The proposal is

> *That the number of images recorded under each unit of the magnitude scale be counted and tabulated.*

The object of the present series of papers is to illustrate the advantages of this simple process, which gives

(*a*) Information as to anomalies of scale at the particular observatory.

(*b*) Information as to the stars themselves and their grouping in space, in such respects as do not need an absolute scale.

2. Under heading (*a*) may be included errors not only of a personal kind, but also systematic errors such as those depending on distance from centre of plate. A dozen years ago it was shown how sensitive the method of counting is in this respect (*M.N.*, lxii. p. 434), and the method as applied at Greenwich showed how differently bright stars and faint stars were affected (*M.N.*, lxxiii. p. 518). A want of symmetry in distribution first noticed by this method in 1908 (*M.N.*, lxviii. p. 409) was traced by Professor Eddington to a tilt in the object glass of a few minutes of arc, thus further illustrating the sensitiveness of the method of counting

(*M.N.*, lxxiii. p. 523). Among errors of a personal kind revealed at once by the method, there have been noticed

(α) Irregularity of scale (all three papers cited).
(β) Decimal equation (*M.N.*, lxix. p. 394).
(γ) Excess of "even" records (*M.N.*, lxix. p. 397).
(δ) Date of measurement (present paper).

3. Under heading (*b*) we have chiefly considered hitherto the question whether the galaxy is richer in faint stars, as Kapteyn claimed for visual magnitudes. It was pointed out in 1909, when the method of counting was proposed to the Permanent Committee in Paris, how definitely the method of counting pronounced against any excess of faint stars for *photographic* magnitudes. This pronouncement was confirmed by the further counts for Bordeaux and Algiers, and has recently been independently confirmed and extended by Chapman and Melotte. It may fairly be taken now as an established fact, but the evidence offered in 1909 only served to discredit the method with those who were prejudiced in favour of a different result, and accordingly no support for the above simple proposal was forthcoming. Instead of it, an elaborate procedure for determining the absolute scale at each observatory was voted: a project which seems bound to fall by its own weight.

4. In place of the discredited richness of the galaxy there is now emerging, from the counts already tabulated, a fact of an equally fundamental kind which seems hitherto to have escaped attention. In the paper on the Bordeaux magnitudes it was remarked in the summary (*M.N.*, lxxii. p. 473):

§§ 5–8. There is also a systematic excess of bright stars near R.A. 12^h.
§ 9 which is four times Kapteyn's galactic effect.

At the time some natural hesitation was felt in ascribing this to a real physical cause, and it was discussed whether it might not be due to temperature or some seasonal effect. But comparison of other results points definitely to its being real. The existence of a similar systematic change in R.A. is illustrated clearly in the results which form the basis of the present paper, but it will be preferable to reserve the discussion of the whole series until the Perth results, which are practically ready, have been presented.

5. We proceed now to tabulate the results of the counts in a form as nearly similar to that for Bordeaux and Algiers as the different practices of the observatories allow. At the Cape the diameters of the star images were measured for the brighter stars, but the faintest images were classified under the headings -1, -2, -3, -4, -5. The results of the counts are given in Table I.

TABLE I.

Number of Images recorded under certain headings in the Catalogue of the Cape −41°.

R.A. h	200 and over.	170–199.	150–169.	130–149.	110–129.	100–109.	80–99.	60–79.	−1.	−2.	−3.	−4.	−5.	Total.
0	42	24	25	46	69	83	90	65	81	81	125	119	91	941
1	33	16	30	48	61	63	71	69	50	59	117	169	146	932
2	42	12	27	44	66	60	63	57	47	39	92	139	301	989
3	69	13	21	61	59	36	68	60	28	44	84	147	278	968
4	48	24	23	27	74	115	148	107	50	47	101	161	312	1237
5	104	45	24	35	155	341	361	233	108	101	183	295	745	2730
6	96	38	47	101	199	195	249	258	161	158	357	516	307	2682
7	201	83	73	120	444	534	627	468	173	233	421	711	1515	5603
8	318	95	111	242	491	411	470	399	232	402	961	305	397	4834
9	156	63	89	211	400	292	333	359	240	443	750	870	951	5157
10	85	33	29	92	229	334	325	380	193	243	487	975	642	4047
11	98	28	29	49	156	283	181	222	142	144	274	407	961	2974
12	63	43	41	91	156	204	226	238	133	180	458	594	194	2621
13	66	23	27	90	251	218	202	206	148	203	319	644	814	3211
14	66	18	33	76	186	215	266	247	125	208	427	826	1228	3921
15	76	17	35	112	237	271	275	279	182	292	507	1021	1331	4635
16	105	28	50	131	219	241	209	311	180	187	336	637	1698	4332
17	109	33	48	136	482	699	566	502	296	572	810	1201	2607	8061
18	68	62	50	101	257	448	354	304	568	897	683	21	12	3825
19	49	29	22	57	168	166	299	297	343	499	129	182	76	2316
20	41	27	25	55	112	139	162	115	150	164	355	153	155	1653
21	31	14	11	23	79	117	101	87	49	36	95	160	451	1254
22	61	16	18	43	81	99	134	115	50	42	112	161	483	1415
23	73	32	24	38	91	97	85	79	42	28	93	92	321	1095
Sum	2100	816	912	2029	4722	5661	5865	5457	3771	5302	8276	10506	16016	71433
N′	2100	2916	3828	5857	10579	16240	22105	27562	31323	36635	44911	55417	71433	
log N′	3·322	3·465	3·583	3·767	4·025	4·211	4·345	4·440	4·496	4·564	4·652	4·744	4·854	
log (C.M.)	9·43	9·75	10·01	10·46	11·09	11·59	11·95	12·22	12·38	12·57	12·82	13·10	13·45	

6. The grouping shown at the head of first eight columns of Table I. is somewhat arbitrary. The counts were actually made in smaller divisions, but it seems superfluous to give full details here. At the foot of the columns are given:

Firstly, the total number of stars under each head;
Secondly, the cumulative totals N′;

Thirdly, log N′; and

Fourthly, the magnitude on the scale assigned by Chapman and Melotte for the whole sky, assuming that the 144 plates cover an area of 144×4 square degrees. The four square degrees is perhaps not exact enough; a more exact estimate would add or subtract a small constant to log N′, and thus alter the magnitudes by an *approximately* constant quantity. If we calculate the magnitudes from the measured diameters d according to the empirical formula

$$\text{mag.} = 16{\cdot}46 - 0{\cdot}513 \times \sqrt{d}$$

we get results as shown below.

TABLE II.

Errors of Scale.

d.	Formula.	C. & M.	F. – C.M.
200	9·21	9·43	– 0·22
170	9·77	9·75	+ 0·02
150	10·18	10·01	+ 0·17
130	10·61	10·46	+ 0·15
110	11·08	11·09	– 0·01
100	11·33	11·59	– 0 26
80	11·87	11·95	– 0 08
60	12·48	12·22	+ 0·26

The residuals shown in the last column indicate systematic errors of measurement of diameter similar to those found at other observatories. The further investigation of them can be best undertaken at the observatory.

7. Coming to the stars which have no measured diameters but are classified as – 1, – 2, – 3, – 4, – 5, we see at a glance that there is a systematic error which is specially notable at 18^h. Turning to the details for the plates given at the beginning of the volume, it is seen that the plates in 18^h were the first to be measured, and the inquiry naturally arises whether there is a change in method of measurement as time goes on. It was easy to rearrange the counts in order of date instead of in order of R.A., and the suggestion of systematic error depending on date of measurement was at once confirmed. It will suffice here to give the following brief table clearly showing the error: the detailed examination can be carried out much better at the observatory, where the history of arrangements is more fully known.

TABLE III.

Systematic Error depending on Date of Measurement.

Year. / Heading.	1898.	1899.	1900.	1901.	1902.	1903.	1906.	Const. = log m.
−5	−1·09	−0·14	+0·20	+0·45	+0·11	+0·07	+0·38	9·60
−4	−0·49	+0·12	+0·28	+0·07	−0·12	−0·11	+0·26	9·50
−3	+0·13	+0·21	+0·20	−0·08	−0·21	−0·42	+0·17	9·40
−2	+0·46	+0·04	+0·15	−0·14	−0·24	−0·49	+0·23	9·20
−1	+0·39	−0·06	−0·04	−0·08	−0·24	−0·11	+0·15	9·12
All	−0·08	−0·03	+0·12	+0·11	−0·14	−0·19	+0·20	0·17
To 99	+0·03	−0·02	−0·02	−0·02	+0·08	+0·09	−0·13	9·61
100 to 149	−0·01	−0·01	+0·04	+0·02	−0·06	−0·05	+0·05	9·65
150 to 199	+0·03	+0·10	−0·06	−0·08	−0·05	−0·06	+0·12	8·80
over	−0·14	+0·10	−0·02	+0·07	−0·02	−0·16	+0·20	8·86
Const. = log n.	3·55	3·79	3·77	3·84	3·49	3·19	2·81	

8. To form the table the logarithms of the counts under each heading were taken (say log M): from this was subtracted log n, where n is the total number of stars *measured* on those plates (*i.e.* excluding the estimates − 1 to − 5). The result gave log ratios of the individual counts to total count of bright stars. Next, to exhibit the variations of scale clearly, log m was subtracted, being simply the mean value of each row. It will be seen that the numbers in each row add up to zero. Hence if E be an element of the table,

$$\log M = E + \log n + \log m,$$

whence the approximate value of M may be recovered if desired.

But the important point is the variation in the rows as we change the date from left to right. In 1898 the logarithm − 1·09 for heading − 5 shows that we have less than one-tenth of the average number of stars for this heading, while in 1901 we have nearly three times the average (0·45 = log 2·8).

These large changes do not occur in the last four rows, though some of the smaller changes there shown may be worth attention.

The same remark applies to the row initialled "All," *i.e.* all the fainter stars from − 1 to − 5, which seems to show that the total number of stars registered remains satisfactorily consistent, though the fainter ones are assigned to different headings in different years.

9. To prevent misunderstanding, it may be clearly stated that in drawing attention to these systematic errors in the Cape results it is not implied that such errors do not occur elsewhere: quite the contrary. The main contention in this series of papers is that there are many such errors which can be easily made manifest by

the simple method of counting, and can then be investigated at the observatory; and further, that the existence of so many sources of error renders the tabulated results quite unsuitable for investigation by the elaborate processes formulated in the resolutions of the conference of 1909.

10. Returning to the row initialled "All," it appears, as above remarked, that though the individual headings -1 to -5 are affected with so large a systematic error that the scale cannot be used, nevertheless the total numbers of stars estimated are consistent. Apparently the faintest stars recorded are of magnitude about 13·45 on the scale of Chapman and Melotte. I must confess that this figure gave me a start. Those who took an active part in the earlier discussions on the scope of the work will remember the keenness with which it was debated whether the limiting magnitude for the Catalogue plates should be 11·0 or 11·9: those whose acquaintance is more recent may be sufficiently interested to refer to the *Observatory*, xii. p. 438, and xiii. p. 89. Apparently we have gone far beyond even the fainter limit, for though the Cape plates go a little further than most, other observatories are not far behind. Turning back to the foot of Table I., we see that the Cape Observatory has measured 71,000 stars where magnitude 11·0 would have indicated 10,000 and magnitude 11·9 would have meant 22,000. We have gained in one way by the overriding of the resolutions: on the other hand, we have waited twenty-five years for the first-fruits from the southern hemisphere. No doubt these facts are not new; but the various uncertainties about the photographic scale have obscured them a little. In previous papers Kapteyn's visual scale was used, but this may now be discarded in favour of Chapman and Melotte's photographic scale, which, without being necessarily absolute (see *Observatory*, xxxvii. pp. 298 and 320), is a great advance on anything previously available.

11. We may now see what the counts will tell us about the ratio of faint stars to bright in different parts of the zone. We need not go too much into detail, it will suffice to select four points of the magnitude scale, viz. the sums of the rows of Table I. as far as

	Col. 2. $d=170$	Col. 5. $d=110$	Col. 8. $d=60$	Col. 13. All stars
Mag. C. & M.	9·75	11·09	12·22	13·45
Diff.	1·34	1·13	1·23	

The material might be presented in various ways, but we will select the variable log N′, *i.e.* the total number of stars recorded as brighter than the limit in question. As it is our main object to discuss variations, we will subtract the mean value of log N′ for each column from the constituents of the column. Further, we will refer the groups to one of them as standard, and we will select for this purpose the stars to magnitude 9·75, as probably the best

determined, in spite of their smaller number. Thus to get the value of log N′ for any element of Table IV. we must proceed as in the case of Table III.—add to the residual shown the constant at the foot of the column and the number shown for mag. 9·75 on the left. To avoid decimals the logarithms are given in units of 0·01. Thus the value of log N′ for mag. 12·22 at R.A. 9^h is

$$(-2+298+32)/100=3\cdot28.$$

TABLE IV.

Values of Log N′ – Log N_o' for Selected Magnitudes in Units of 0·01.

R.A.	Gal. Lat.	log N_o' to Mag. 9·75.	Residuals for Mags. II. 11·09.	III. 11·22.	IV. 13·45.	III – II.	IV. + III. – II.
0	−76°	−20	− 6	−13	−22	− 7	−29
1	−73	−33	+ 3	− 6	− 9	− 9	−18
2	−64	−29	0	−12	−10	−12	−22
3	−53	−11	−11	−28	−29	−17	−46
4	−42	−16	−12	− 7	−14	+ 5	− 9
5	−31	+15	−16	− 2	−10	+14	+ 4
6	−20	+11	0	− 2	− 7	− 2	− 9
7	− 9	+43	− 4	0	− 7	+ 4	− 3
8	0	+60	− 7	−18	−31	−11	−42
9	+ 8	+32	+ 7	− 2	0	− 9	− 9
10	+15	+ 5	+ 5	+15	+17	+10	+27
11	+20	+ 8	− 9	− 4	0	+ 5	+ 5
12	+22	+ 1	+ 2	+ 4	+ 2	+ 2	+ 4
13	+21	− 7	+16	+12	+20	− 4	+16
14	+17	−10	+11	+17	+30	+ 6	+36
15	+11	− 5	+16	+18	+33	+ 2	+35
16	+ 4	+10	+ 6	+ 3	+15	− 3	+12
17	− 6	+13	+21	+30	+39	+ 9	+48
18	−16	+ 9	+ 7	+15	+10	+ 8	+18
19	−26	−13	+ 7	+19	+10	+12	+22
20	−37	−19	+ 4	+ 4	+ 2	0	+ 2
21	−49	−37	0	+ 6	+ 8	+ 6	+14
22	−60	−13	−10	−10	−11	0	−11
23	−70	0	−17	−26	−35	− 9	−44
Constant		202	257	298	339	...	...
Sum for 10^h – 21^h			+86	+139	+186	+53	+239
Sum for 22^h – 9^h			−73	−126	−185	−53	−238

12. In the second column of Table IV. the galactic latitude has been inserted. It only requires a cursory glance to see that

this has no relation to the residuals. We need not "flog a dead horse" by a detailed examination of this point, for the principal feature of the residuals is plain to the eye, and is emphasised by the thick lines. From $10^h - 21^h$ the residuals are positive, that is, there is a relative excess of faint stars: in the remainder of the zone they are negative. The change from positive to negative is almost abrupt, and reminds us of those photographs where a line of nebula seems to divide the plate into two regions, in one of which stars are more numerous than the other. The three groups support one another very well, the excess showing consistent development as we proceed to fainter stars. The sums for $10^h - 21^h$ and for $22^h - 9^h$ shown at the foot of each column show this progression clearly, and their difference can be closely represented by the linear formula

$$100 \times \text{magnitude} - 960$$

as shown below:

Mag.	9·75	11·09	12·02	13·45
Observed	0	+159	+265	+371
Calculated	+15	+149	+242	+385
O – C	–15	+ 10	+ 23	– 14

The slight curvature indicated may be significant, but the material scarcely warrants our so treating it.

13. Any anomalies in the column of stars to 9·75 will affect all three columns of residuals. The measures for these bright stars are probably the best, but the number of them is small. Again, column IV. is subject to the uncertainties attending the registration of the very faintest stars on the plate. If we subtract column II. from column III. we overpower the bright stars and avoid the errors attending the faintest. Hence though the interval of magnitude is very much smaller, column III. – II. may be put alongside column IV. (which is of course IV. – I.) as a totally independent check on any systematic variation; and if we add the two together with equal weights, we reduce some of the accidental errors. The abruptness of change from negative to positive is, however, by no means eliminated, even in column III. – II. alone. We can exhibit this abruptness by writing the numbers near the junctions as below, reversing the negatives:—

Begin 10^h	+27	+ 5	+ 4	+16	+ 36	+ 35
Turn back 21^h	+14	+ 2	+22	+18	+ 48	+ 12
Begin 22^h	+11	+44	+29	+18	+ 22	+ 46
Turn back 9^h	+ 9	+42	+ 3	+ 9	– 4	+ 9
Sum	+61	+93	+58	+61	+102	+102

If the residuals made a sinuous curve, these numbers should gradually fall to zero as we proceed from right to left. Clearly they

do not do so, and the hypothesis of an abrupt change is more suitable than that of a sinuous curve.

14. Nevertheless we may calculate the harmonics in the usual way for columns II., III., and IV. The results are :—

II. $10{\cdot}2\cos(\theta-237^\circ)+3{\cdot}1\cos 2(\theta-253^\circ)+3{\cdot}1\cos 3(\theta-270^\circ)+4{\cdot}6\cos 4(\theta-309^\circ)$

III. $16{\cdot}4\cos(\theta-239^\circ)+6{\cdot}4\cos 2(\theta-274^\circ)+2{\cdot}0\cos 3(\theta-303^\circ)+2{\cdot}4\cos 4(\theta-291^\circ)$

IV. $21{\cdot}4\cos(\theta-233^\circ)+7{\cdot}0\cos 2(\theta-256^\circ)+1{\cdot}2\cos 3(\theta-305^\circ)+2{\cdot}0\cos 4(\theta-328^\circ)$

The third and fourth harmonics show no progressive increase in the coefficient, and it may be doubted whether they are real. But their phases show a certain accordance with the others, and the arbitrary constant has been used to exhibit this as plainly as may be. By the arbitrary constant is meant the 360°, 180°, 120°, or 90°, which may be added to the phases of the 1st, 2nd, 3rd and 4th harmonics. Taking the means of those above shown, we get

$$236^\circ \qquad 261^\circ \qquad 293^\circ \qquad 309^\circ$$

which suggests an arithmetical progression. But there is scarcely enough material to warrant any inference.

15. Indeed, if these results stood alone, it might be objected that we have already treated them too seriously. But they do not stand alone: it was pointed out in discussing the Bordeaux results (*Mon. Not.*, lxxii. p. 468) that there is a considerable first harmonic in the ratio of faint stars to bright, so considerable that at first sight it seemed possibly an instrumental or seasonal effect; and one or two tests were made in this direction, without, however, finding any evidence of such causes. The Algiers results (*Mon. Not.*, lxxii. p. 702) also show a sensible first harmonic, the coefficients of sin α and cos α in Table IV. changing slowly for successive magnitudes; but the changes are certainly less striking. In the Cape results they are considerable, and other series seem to be similarly affected. But the evidence will be more conveniently collated when one or two other series (especially the Perth results for zone -31°, which have just appeared, and the Vatican results for $+64^\circ$ now in the press) have been presented. A first glance at them suggests a division of the whole sky into two regions of greater and less absorption of light, the boundary having little or no relation to the Milky Way.

Summary.

§§ 1–4. General remarks on the simplicity and sensitiveness of the method of counting, which has already detected various systematic errors.

§ 5. Presentation of the counts for -41°.

§ 6. Comparison of the Cape scale with that of Chapman and Melotte, and with the formula $a - b\sqrt{d}$.

§§ 7–10. The estimates for faint stars are affected by a con-

siderable systematic error depending on date of *measurement* of the plate, but the total numbers of stars on the plates are consistent.

§§ 11–15. The ratio of faint stars to bright is much greater for one half the zone than for the other. Other zones show similar results which will be studied together when one or two more zones have been presented.

The Transit of Mercury, 1914 *November* 6–7.

By the Rev. A. L. Cortie, S.J.

The observations were made at the Stonyhurst College Observatory, the co-ordinates of which are $\phi = 53^\circ\ 50'\ 40'', \lambda = 9^m\ 52^s{\cdot}68$ W. The egress was missed owing to clouds, but the ingress was observed in fairly good definition by projection with the 15-inch equatorial, the size of the solar image being 10·5 inches in diameter. The following are the times:—

External contact at ingress—

	h	m	s	
First distinct appearance . .	21	58	4·0	G.M.T.

Internal contact at ingress—

Geometrical contact . . .	21	59	39·5	G.M.T.
Formation of the black drop . .	21	59	49·5	,,
Black drop most developed . .	22	0	2·0	,,
Distinct separation of limb . .	22	0	12·0	,,

These observations were made by Mr. William McKeon. Judging by discussions of the transits of Venus, it would seem that the most likely instant of internal contact was when the black drop was most developed and was on the point of breaking.

During the transit of the planet across the disc of the Sun, it was carefully examined by Mr. M'Keon and myself for any signs of an atmospheric ring, with negative results. These observations were made not only by projection of the Sun's image, but also directly by means of a Thorp polarising solar eye-piece, using a magnification of 100. The planet was also examined by us spectroscopically with the Browning solar spectroscope, using a dispersion of six prisms of 60°. The slit was placed parallel to the diurnal motion. The definition was very good. The planet appeared very like the umbra of a very black Sun-spot. There was, however, no penumbral fringe, but a clear-cut very dark band across the spectrum. Nor was any sign of a penumbral fringe observable when the planet was brought by the slow motion in declination just off the slit.

Stonyhurst College Observatory:
1914 *Nov.* 23.

Observations of the Transit of Mercury on 1914 *November* 6–7, *made at the University Observatory, Warsaw.* By Professor S. D. Tscherny.

1. The observations were made by Director Professor S. D. Tscherny and S. A. Swiderski.

2. *Instruments.*—Professor Tscherny: Refractor by Merz, aperture 162 mm., focal length 265 cm., eye-piece magnifying power 94.

S. A. Swiderski: Refractor by Merz-Sabler, aperture 102 mm., focal length 141 cm., eye-piece magnifying power 93.

3. Comparison of the chronometers Nardin 498 (mean time) and Dent 1630 (sidereal time), with Hardy sidereal clock:

h	m	s			h	m	s		h	m	s	
23	4	55·92	Nardin 498 =		14	6	32·23	Dent 1630 =	14	6	32·00	Hardy.
2	54	4·83	Nardin 498 =		17	56	18·84	Dent 1630 =	17	56	18·55	Hardy.

4. *Error of Hardy Sidereal Clock.*—Five stars (θ Lyræ, δ Aquilæ, β Cygni, θ Cygni, and τ Draconis) were observed by Professor Tscherny with the Ertel Meridian Circle, and gave a clock error on Warsaw sidereal time $-10^{s}\cdot486$ at $19^{h}\ 24^{m}$ Hardy on November 7. Rate of Hardy $-0^{s}\cdot290$ per day (24 hours).

5. *Error of Chronometers on Warsaw Time.*—From the above comparisons and observations the errors of chronometers were found to be—

	m	s		h	m	s	
Nardin 498:	−1	33·72	at	23	4	56	Nardin 498
Dent 1630:	−0	10·66	at	14	6	32	Dent 1630
Nardin 498:	−1	33·78	at	2	54	5	Nardin 498
Dent 1630:	−0	10·76	at	17	56	19	Dent 1630.

6. *Observations of Contacts.*—Professor Tscherny's observations by Nardin 498:—

		h	m	s			h	m	s
I.	Contact	23	24	1	a little late.	Warsaw mean time	23	22	27·28
II.	,,	23	25	39	very good.	,, ,, ,,	23	24	5·28
III.	,,	3	31	42	the limb was boiling.	,, ,, ,,	3	30	8·22
IV.	,,	3	33	48·5	satisfactory.	,, ,, ,,	3	32	14·72

Swiderski's observations by Dent 1630:—

III.	Contact	18	33	52	the limb was boiling.	,, ,, ,,	3	29	58·06
IV.	,,	18	36	2	good.	,, ,, ,,	3	32	7·70

Mercury was remarkably black compared with the Sun-spots in N.E. of the Sun, quite round, and sometimes fairly well defined. During the whole time I saw no white spot on the planet or anything to mar its blackness, and there was no surrounding ring of light. Also I did not see the "black drop."

7. *Results of Warsaw Observations of Contacts.*

	Obs.	Nov. 1914.				C.d.T.		O–C.	N.A.	O–C.	A.E. & N.A.	O–C.
		d	h	m	s	m	s	s	s	s	s	s
I.	1	6	23	22	27·28	21	12·6	+74·68	10·55	+16·73	10·25	+17·03
II.	1	6	23	24	5·28	23	25·6	+39·68	24·85	−19·57	23·75	−18·47
III.	2	7	3	30	3·14	30	56·8	−53·66	43·05	−39·91	44·42	−41·28
IV.	2	7	3	32	11·21	33	10·1	−58·89	55·45	−44·24	58·19	−46·98

8. *Observations with Boguslawski's Micrometer.* — Professor Tscherny observed the moments of passage across the border of lamina of both borders of Mercury and eastern border of the Sun. The lamina had the direction of the *true* declination circle. These moments are as follows :—

Centre of Mercury.	Chronom.	Eastern Border of Sun.
h m s		h m s
23 45 49·5	Nardin 498	23 46 38
23 47 14·5	,,	23 48 3·2
23 48 44·5	,,	23 49 34
23 50 18·5	,,	23 51 9
23 51 37·5	,,	23 52 28
23 53 7·5	,,	23 53 58
23 54 37·5	,,	23 55 29
23 56 27·5	,,	23 57 20
23 57 55·5	,,	23 58 48
23 59 21·75	,,	0 0 15
0 0 58·5	,,	0 1 52
0 2 52·5	,,	0 3 47
0 4 35·5	,,	0 5 30·5
0 7 18·5	,,	0 8 15
0 9 9·5	,,	0 10 6
0 10 35·5	,,	0 11 33
0 12 1·5	,,	0 12 59·5
0 13 3·35	,,	0 14 32
0 15 4·35	,,	0 16 3·2
0 16 50·5	,,	0 17 51
0 18 32·45	,,	0 19 33
0 20 22·5	,,	0 21 23·5
0 23 4·3	,,	0 24 6·1
0 25 27·45	,,	0 26 31
0 27 4·45	,,	0 28 7·6
0 29 16·4	,,	0 30 20·2
0 30 55·5	,,	0 32 0·9
0 33 11·45	,,	0 34 17
0 38 23·5	,,	0 39 31
0 39 57·5	,,	0 41 6
0 41 35·5	,,	0 42 44

Warsaw University Observatory :
1914 *November* 14.

The Temperature Coefficients of the Edinburgh Transit Circle. By Professor R. A. Sampson and E. A. Baker, B.Sc. (Plates 5–7.)

The remarks made by Gill at the meeting of 1913 January regarding the use of fixed terrestrial azimuth marks with the Cape Transit Circle deserve attention.* A full description of the results has now been published in the *Cape Annals*, vol. xi. part iii. (1914). Azimuths determined from these marks have proved so reliable that by comparison with stellar observations even the variation of latitude, or rather the complementary lateral polar deviation, may be exhibited. The existence of these marks rendered possible Mr. Hough's scrutiny of the periodic errors in R.A. of the Catalogues of Newcomb and Boss (*M.N.*, lxxiii. 3, 1913 January).

Marks of this kind are not easy to set up. To sink the mark itself to such a depth in the rock that it should remain fixed secularly might give comparatively little difficulty, but provision of lenses of very long focus—those at the Cape are of 300 feet—would certainly involve cost, trouble, and delay. This lens is an essential part. It is apparent that it also must be adjusted over a deep fixed mark, as is done in the Cape equipment, since any lateral movement (x) of its mounting will shift the zero adopted for the azimuth measures by x/f where f is the focal length of the lens. The arrangement seemed to deserve further study. No one would use a 300-foot telescope with lens and focal marks entirely separate from one another if equivalent results could be otherwise obtained. It seemed possible that the ordinary collimating telescopes might give valuable results. Some recent work at Edinburgh has been connected with a scrutiny of the accuracy of time determinations, and in this naturally the study of instrumental error of the Transit Circle has played a large part. This is the subject of which the following paper gives an outline.

Some provision has been made in the design of the South Collimator of our Transit Circle for the use of an azimuth mark. The pier which carries it forms a granite table in which two iron supports ending in V's are embedded. These supports are made of sufficient height and are pierced with apertures which would permit the Transit Circle to be pointed so as to look straight through them in a line which strikes the ground about 150 feet away. The cell of an object-glass could have been attached to one of these apertures, and thus an azimuth mark upon Gill's plan could have been provided without any material disturbance of the existing arrangements. In view, however, of the difficulties mentioned above, it seemed best to begin at any rate by taking the collimators themselves as azimuth telescopes, for it appeared quite possible that their greater compactness, and all the control that went with it, might counterbalance a large part of the loss due to their small focal length of about $6\frac{1}{2}$ feet and less absolute fixture.

* *Observatory*, No. 458 ; a description of the marks is given in *Observatory*, No. 459, and more fully in Gill's *History of the Cape Observatory*, p. 38.

The observations differ only in detail from those habitually made. The movable wire of the North Collimator was set in contact right and left five times with the image of the fixed vertical wire of the South Collimator, giving a mean reading [N/S] to which the North Collimator was then set; then similar settings of the Transit Circle on the North and South Collimators [TC/S and TC/N] were made, and for level the mercury trough with Bohnenberger eye-piece give a reading [L] for coincidence of images; then the quantities discussed below are

$c = constant - \frac{1}{2}[\mathrm{TC/S} + \mathrm{TC/N}] = \text{collimation.}$
$l = \mathrm{L} - \frac{1}{2}[\mathrm{TC/S} + \mathrm{TC/N}] = \text{level.}$
$\mathrm{A} = constant + \frac{1}{2}[\mathrm{TC/N} - \mathrm{TC/S}] = \text{azimuth of Transit Circle east of South Collimator.}$
$[a] = constant - \mathrm{N/S} = \text{azimuth of North Collimator east of South Collimator.}$

The factor for converting c, l, A into seconds of time is $1^{\mathrm{rev.}} = 1^{\mathrm{s}}{\cdot}313$; the corresponding factor for $[a]$ is $1^{\mathrm{rev.}} = 1^{\mathrm{s}}{\cdot}824$. Thus, without adding sensibly to the work, measures are obtained of the azimuth of the Transit Circle with respect to the collimators and of the collimators with respect to one another. The measures are, indeed, relative only, but so are all measures, and it is only by external comparisons and often by elaborate criticisms we can judge what absolute reliance may be placed upon any measure. In the present case the comparison takes the form of a collation with celestial azimuths, which are themselves relative to a number of things, in particular to the difficulties of observing polars, to Catalogue errors, and to movement of the earth's pole.

The observations—except those expressly for stellar azimuths—were begun on 1913 October 1 by Professor Sampson and continued by him until 1914 March 16, after which date they were taken by Mr. Baker. It was intended that they should furnish as far as practicable continuous data of the behaviour of the instrument as contrasted with the detached information available when similar observations are made only in order to reduce stellar transits; with this object they were made as often as practicable each day; the number made has seldom been less than three in the day, and since March, with few exceptions, it has been four, namely at 1, 7, 11, and 22 hours. Temperature has been read at the same time. At first several thermometers were read and considered—external, on the collimator piers and on the transit piers, but only the last is now retained and the temperatures given below refer to it. It soon became apparent that both level and azimuth thus derived followed the temperature of the piers in a general way, and often, more than this, in surprisingly close detail. Alongside this and often masking it there was visible a basic fluctuation of the standard from which these superficial temperature changes were reckoned. Thus the material presented itself chiefly as a study of the temperature coefficient, superficial and basic, of the different errors of the

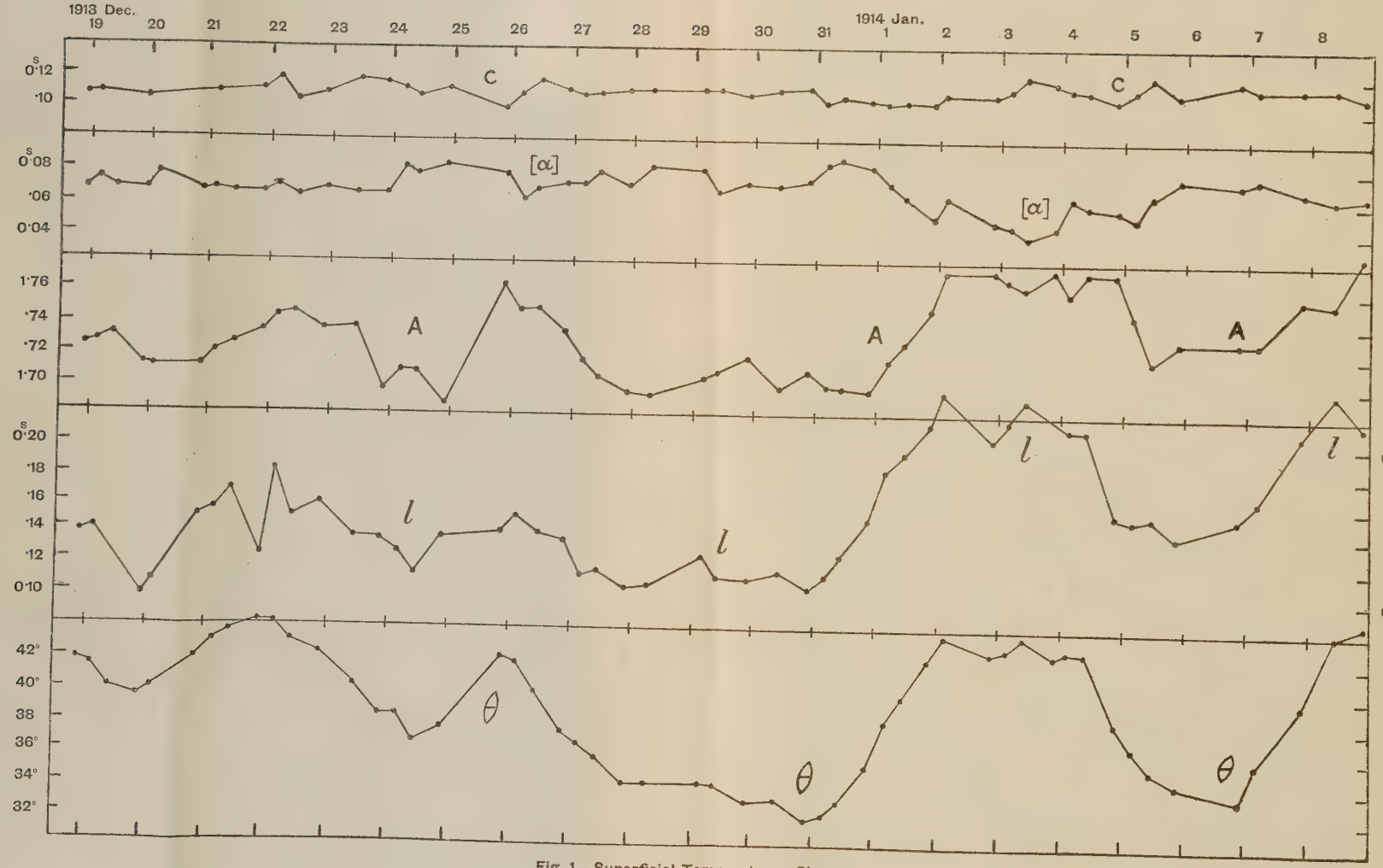

Fig. 1.—Superficial Temperature Changes.

Transit Circle, and from this point of view it is treated. Naturally it will not be expected that a complicated instrument should strictly and always follow a single thermometer placed in any one position. The results are regarded as preliminary. It is proposed to continue the inquiry to ascertain how far the features disclosed repeat themselves in another year, and if possible to assign more closely the laws which they follow.

The observations themselves are too numerous to give here except in extract. Fig. 1 shows the observations of three consecutive weeks, 1913 December 19–1914 January 9. Broadly speaking, the three lowest curves—showing azimuth, level, and temperature—are identical. One might in particular compare l and θ over the interval December 29^d 10^h–Jan. 5^d 4^h. The accordance of the azimuth curve is hardly less striking. It will not weaken this evidence that at other portions of the traces an agreement cannot be traced so readily. The failure never persists for long, and after it the parallelism is re-established. The conclusion from an examination of the whole is that there are superficial temperature coefficients in azimuth and level, declaring themselves without sensible lag, obscured at intervals by other changes which may be termed the basic changes. The systematic consideration of these is outlined below.

With respect to collimation, we cannot say that the section shown exhibits any definite evidence of a relation to the thermometer of the Transit Circle pier. There is, however, some indication of a daily period which is examined further below. It will be noticed that this period shows itself as on December 22 and January 5, where in the former case there is only a very slight indication of a corresponding wave in θ, and in the latter case none at all.

The curve $[\alpha]$ of azimuth of the North Collimator east of the South Collimator shows signs of a diurnal period, and also of a temperature coefficient related to θ, the latter of sign opposite to that which affects A and l.

These are the features suggested for further inquiry by a preliminary examination of the curves.

Azimuth Curves.

The first problem to examine was the permanence and constancy of the superficial temperature changes visible above. This was taken in two steps, first graphically and then numerically. As it was clear that the observations could not all be collected in one group because of the basic changes, they were collected in 7-day groups, and each group was plotted with temperature and indicated azimuth as co-ordinates. Three of these groups are shown in fig. 2; these are examples of cases where one can read the apparent gradient with reasonable confidence; there were other cases, especially when the range of temperature was small, where no trustworthy deduction could be made. Nevertheless the whole

of the observations up to the end of May were charted in this way and the gradients read as well as possible, with the following results:—

Gradients for Azimuth, Estimated Graphically.

1913.	Range of Temp.	Gradient per 1° F.		Range of Temp.	Gradient per 1° F.
	°			°	
Oct. 5–11	5·5	+·0108	Feb. 1–7	6·3	+·0072
12–18	6·5	100	8–14	6·1	50
19–25	18·1	84	15–21	9·7	64
26–32	6·6	+ 133	22–28	9·8	68
Nov. 2–8	9·1	− 10	Mar. 1–7	6·1	86
9–15	6·0	+ 118	8–14	9·8	67
16–22	9·1	113	15–21	7·3	+ 112
23–29	7·0	126	22–28	4·8	− 50
30–37	16·3	83	29–35	6·8	+ 106
...	...	...	Apr. 5–11	5·6	102
Dec. 14–20	8·9	86	12–18	10·5	73
21–27	10·3	63	20–25	14·0	81
28–32	10·4	68	26–32	11·5	90
Jan. 2–8	11·6	67	May 3–9	9·6	58
12–16	1·6	...	10–16	12·4	83
18–24	9·8	40	17–23	7·9	120
25–31	10·3	+·0044	24–30	11·5	+·0084

The mean of these is $+^{s}\cdot0076$ per 1°. This was considered as fairly well established, and it seemed plausible to adopt it as a first approximation as constant throughout the set. It was then used to find the basic curve as follows:—Each observed azimuth was reduced to temperature 45° by applying a correction with coefficient ·0076 per 1°, and the results meaned in weekly groups. This gave a basic azimuth curve supposed clear from superficial temperature effect. This is the curve A_0 of fig. 3. Its regularity, and also its agreement with the basic level curve, l_0, which was similarly derived and is shown above it upon the same figure, are considered strong presumptions for its essential reality. Any permissible alteration in the adopted temperature gradient would not change it materially. This curve then was accepted, and was employed to make a second, purely numerical, estimate of the temperature gradient from each week's observations. This was done by interpolating the value of A_0 for the time of each observation, and attributing the difference $A - A_0$ between the observed and basic azimuths to the difference of temperature between 45° and that of the observation. The following table shows the results, and also the mean discrepancy or residue for each week between the observed azimuth at any time and the calculated azimuth, starting from the basic azimuth supposed known, and using the temperature gradient indicated for the week.

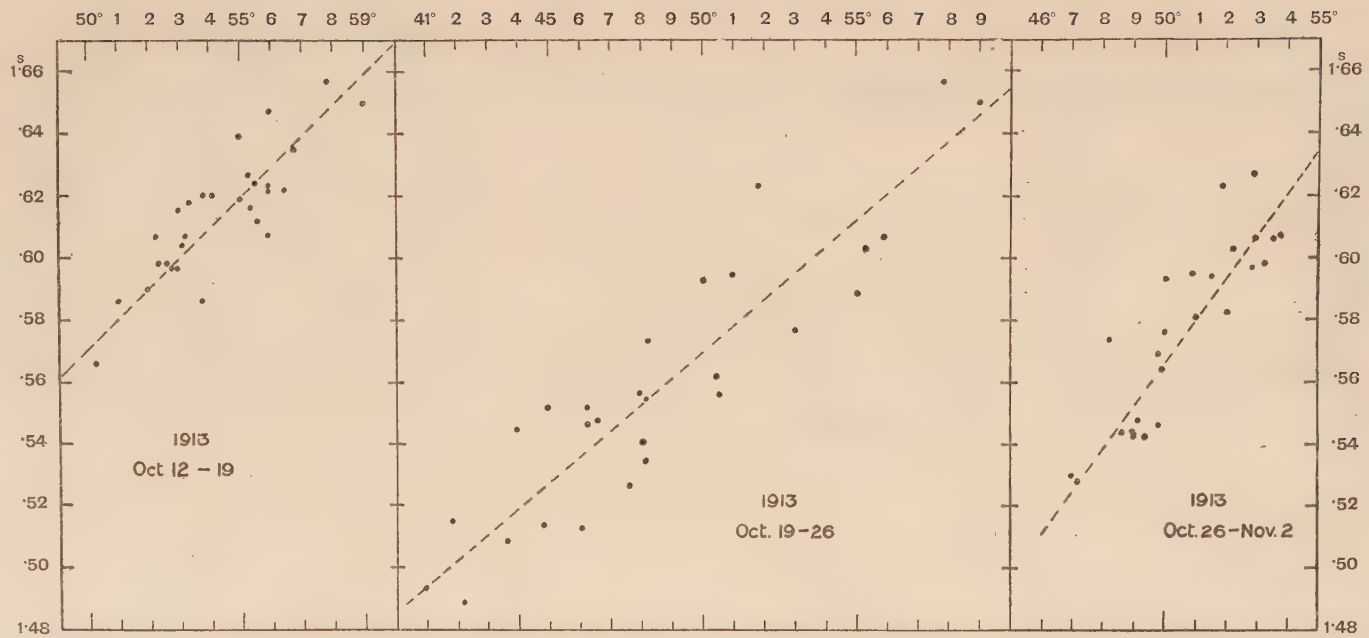

Fig. 2.—Superficial changes; Instrumental Azimuth (vertical co-ordinates) and Temperature (horizontal co-ordinates).

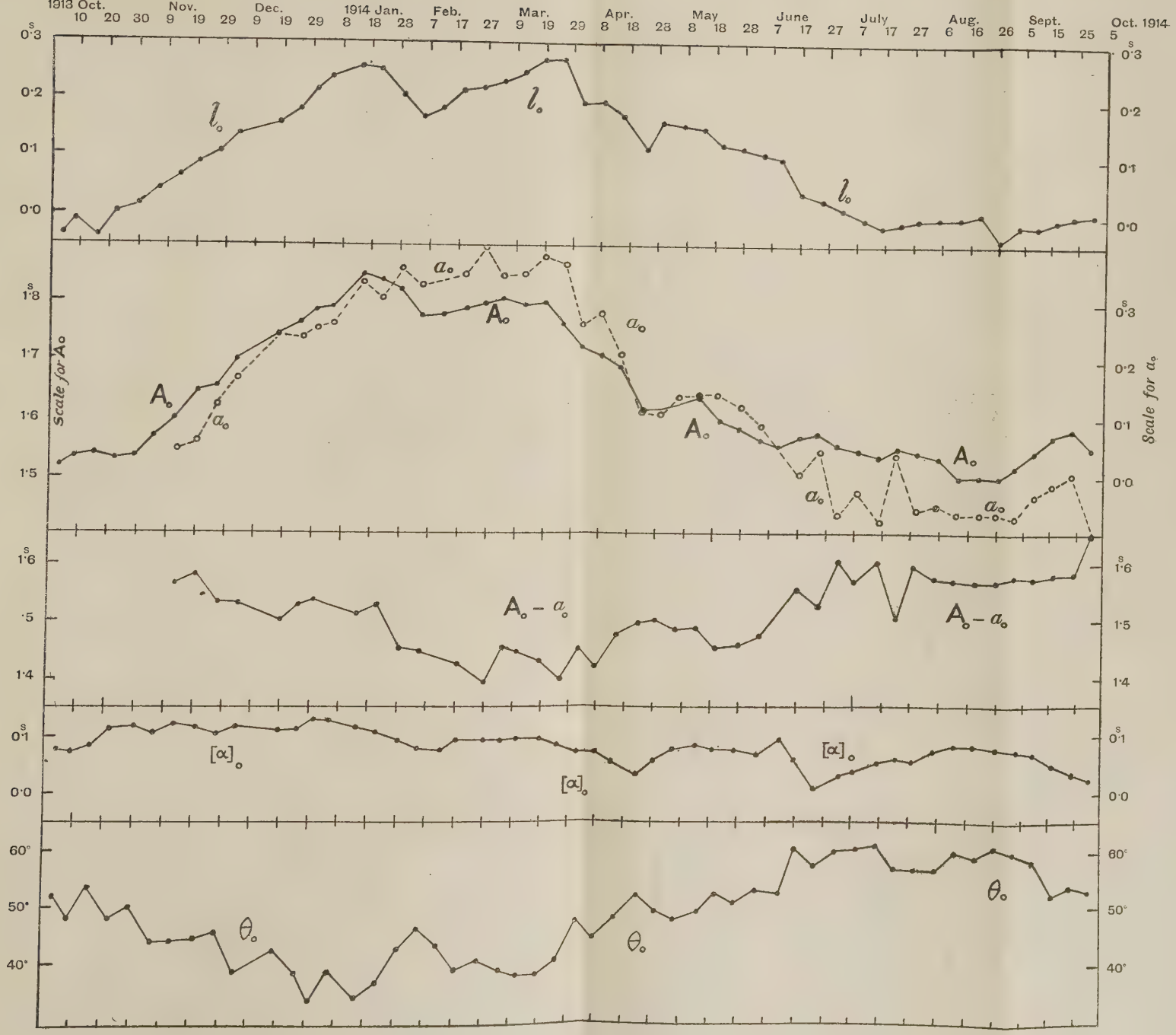
1913 Oct. 10 20 30 Nov. 9 19 29 Dec. 9 19 29 1914 Jan. 8 18 28 Feb. 7 17 27 Mar. 9 19 29 Apr. 8 18 28 May 8 18 28 June 7 17 27 July 7 17 27 Aug. 6 16 26 Sept. 5 15 25 Oct. 5
l_o
A_o
a_o
Scale for A_o
Scale for a_o
$A_o - a_o$
$[\alpha]_o$
θ_o

Fig. 3.—Basic Changes.

Azimuth Gradient, Estimated Numerically, and Mean Residues.

Date.	Gradient.	Residue. s	Date.	Gradient.	Residue. s
Oct. 5–11	+·0056	±·008	Apr. 5–11	+·0068	±·012
12–18	94	·008	12–18	48	·018
19–25	79	·013	20–25	49	·020
26–32	146	·016	26–32	83	·020
Nov. 2–8	19	·009	May 3–9	58	·006
9–15	98	·012	10–16	79	·013
16–22	141	·010	17–23	86	·010
23–29	99	·010	24–30	82	·014
30–37	50	·014	31–37	55	·014
...	...	...	June 7–13	44	·014
Dec. 14–20	63	·009	14–20	16	·014
21–27	61	·011	21–27	3	·010
28–32	36	·008	28–34	80	·012
Jan. 2–8	+ 45	·011	July 5–11	54	·011
12–16	− 13	·008	12–18	100	·011
18–24	+ 37	·008	19–25	80	·013
25–31	50	·009	26–32	60	·009
Feb. 1–7	23	·006	Aug. 2–8	122	·008
8–14	38	·004	9–15	44	·010
15–21	73	·009	16–22	70	·012
22–28	47	·007	23–29	72	·015
Mar. 1–7	61	·013	30–36	49	·016
8–14	52	·008	Sept. 6–12	56	·011
15–21	+ 69	·010	13–19	32	·008
22–28	− 7	·012	20–26	68	·005
29–35	+·0068	±·013	27–33	+·0083	±·013

The mean gradient shown is $+^{s}\cdot0062$ per 1° F.; this may be taken as more reliable than $+^{s}\cdot0076$ per 1° derived by graphical estimation. Returning now to the basic azimuth curve A_0, we see that it shows a maximum in the second week in January, another early in March, and a minimum in the middle of August. It is much smoother than the mean temperature curve (θ_0) which is shown for the same dates on the same figures, but all the main fluctuations of the one are shown in the other, without lag. It seems probable that it would agree with the temperature a little way below the ground or within the pier. This must be reserved for further examination. If so we should have to attribute changes in A to two curves, one which followed the temperature of the metal work of the V's, and a second which followed the temperature within the granite pier or the rock. It will be noticed that the coefficient for the latter change is of opposite sign to that for the former and of about twice its magnitude.

Relation to Celestial Azimuth.

The azimuths read from the azimuth marks of the Cape Transit Circle appear to be of so absolute a character that their smoothed values are more fixed than stellar azimuths themselves determined from polars, since the latter must be cleared of the small fluctuation due to motion of the earth's pole before an even comparison between the two can be made. With this fact in remembrance, special pains have been taken with the stellar azimuth determination for comparison with the curve A. More will be said upon this point below. At the moment attention is drawn to the result of the comparison, which is shown in the second curve of fig. 3. In this A_0, as explained above, denotes azimuth read with respect to the South Collimator, reduced to temperature 45° by an adopted common coefficient $+^s\cdot0076$ per 1° F., and meaned for each week. Superposed to A_0, the stellar azimuth, a_0, is shown connected by a dotted line; this is also meaned for the week and reduced to the same temperature with the same coefficient. The coefficient may be somewhat at fault, but if no reduction at all had been made the comparison of the two curves would have been very much the same. It shows a well-marked wave exhibited below as $A_0 - a_0$. If the determinations of A_0 were absolute and those of a_0 completely reliable, we should expect a level straight line, modified only by the barely sensible movement due to the motion of the pole itself. The fluctuation actually shown is fully $0^s\cdot15$ between its greatest and least values, following to some degree the mean temperature curve. This calls for explanation. Since A_0 is taken with respect to the South Collimator, we might suppose that the optic axis of the South Collimator was liable to movement to this amount. We can only test this by another reference. The fourth curve on the same figure $[a_0]$ shows the mean azimuth of the North Collimator east of the South, reduced to 45° with a coefficient $-^s\cdot0036$ per 1° F. This does not seem to offer any promise of explaining the curve $A_0 - a_0$. If we attribute the latter to the South Collimator, we should be obliged to attribute to the North Collimator, roughly speaking, an equal and simultaneous movement. This appears improbable, as none of these movements show the lag with respect to surface temperature that one would expect from deep-seated movements. There is so little resemblance between the curves $[a_0]$ and $A_0 - a_0$ that one is disposed to conclude that the South Collimator is almost completely steady, and that most of the movement shown in $[a_0]$ is due to the North Collimator. One must then examine the stellar azimuths a_0. We may record as a specimen the details for the interval between March 9 and April 25. In this interval A_0 on the whole falls by $0^s\cdot17$; but a_0, besides erratic movements, falls much more rapidly, *i.e.* by $0^s\cdot22$ at least. The following table shows the observer, the individual stars observed, with the azimuth as deduced from each entered below it, the mean azimuth for the night, (a) the mean for the week, $((a))$ and the same reduced to temperature 45°, a_0 for the

purposes of the chart. It shows also A, for each night as read from the collimators.

Observed Azimuths, 1914 *March* 9–*April* 25.

Date.	Observer.	Temperature.	δ Urs. Min. S.	Cephei 51.	λ Urs. Min. S.	Groomb. 1119.	Groomb. 3548 S.	Brad. 3147 S.	Brad. 1672.	(*a*).	((*a*)).	a_0.	A.
			s	s	s	s	s	s	s	s	s	s	s
Mar. 9	J.S.	34°8	·266	·313	·302					·294			1·723
10	W.	35·4	·330	·285	·344	·289	·364			·322			·727
11	J.S.	36·3	·184	·259	·175	·263				·220			·730
12	W.	40·3		·309	·382	·296				·329			·744
14	W.	43·5				·332	·427			·380	·301	·351	·775
16	W.	40·0	·352	·328	·364	·268	·362			·335			·762
17	J.S.					·289	·200			·245			
18	W.	38·5		·302	·377	·348	·382			·352			·752
19	J.S.	38·3		·275	·255	·319				·283			·726
21	J.S.	39·6		·382	·414	·364				·387	·330	·383	·754
24	W.	42·6		·318	·407	·370	·386			·370			·749
27	J.S.	42·4	·295	·286		·339				·307	·343	·370	·741
29	W.	45·0		·291	·357					·324			·745
31	J.S.	50·0				·305	·310			·308			·776
Apr. 1	W.	51·3		·236		·329	·354			·306			·768
2	J.S.	48·9		·310						·310			·742
3	W.	49·5				·212	·289			·251			·733
4	J.S.	48·0					·184			·184	·289	·268	·718
7	W.	42·7				·171	·219			·195			·686
8	B.	44·0					·264	·262	·292	·273			·709
9	J.S., B.	45·2				·284		·366		·325			·699
10	J.S., B.	47·0					·369	·307		·338			·736
11	B.	46·9				·288	·333			·311	·287	·289	·733
13	J.S., B.	46·2					·269	·192	·287	249			·733
14	J.S., B.	45·6					·204	·186		·195			·707
15	B.	49·1					·234	·366	·335	·312			·688
16	B.	52·8					·233	·150	·244	·209			·704
17	B.	52·6					·186	·145	·272	·201			·725
18	B.	52·9					·303			·303	·240	·213	1·735
20	B.	55·0						·105	·203	·154			...
21	B.	60·8						·207	·281	·244			1·687
23	B.	49·6							·158	·158			·656
25	B.	51·5						·100	·164	·132	·177	·116	1·641

Stellar azimuths must be considered at present the ultimate appeal; yet it is difficult to draw any definite conclusion from the material here before us as to the source and reality of a relative change of a_0 with respect to A_0 of amount $0^s{\cdot}05$ or thereabouts. We may compare the transits of the three stars *Cephei* 51, λ *Urs. Min.*, and *Groomb.* 1119 on March 11 and 12, and there are many other anomalous instances. Some of this may be due to personality in the observers; this is reserved for fuller subsequent examinations. The transits are taken by shifting the wire and tapping the moment of transit on the chronograph. But personality would not generally affect very much the means for the weeks, which are irregular by amounts of the same order as that under discussion. One would be disposed to attribute a considerable part of the separate differences between A_0 and a_0 to uncertainty as to the true value of the latter, but this is an insufficient explanation, since the curve $A_0 - a_0$ does not consist merely in irregularities; there is also a systematic dip in it to the total amount of fully $0^s{\cdot}15$, and this seems very difficult to account for.

Since an error—Δa—in the value adopted for a would reproduce itself in the adopted clock error to amount of the mean value for the whole set of clock stars of Δa $(\sin\phi - \cos\phi \tan\delta)$, which may be about four-fifths of the full value of Δa, some light may be thrown upon the reliability of a by collecting the adopted clock errors for the same period. These are given below for the clock Riefler. Over the same period a nightly comparison was made by wireless with the Paris clock; the result of this is shown alongside the other as *P. – Ed.*

Date.	Riefler's Error.	*P. – Ed.*	Date.	Riefler's Error.	*P. – Ed.*	Date.	Riefler's Error.	*P. – Ed.*
	s	s		s	s		s	s
Mar. 9	+·20	+·16	Mar. 27	+·31	+·05	Apr. 11	+·07	+·16
10	·24	·13	29	·17	...	13	·12	·14
11	·32	·11	31	·06	·08	14	·15	...
12	·24	·07	Apr. 1	+·04	·15	15	·05	·25
14	·25	·13	2	−·01	·21	16	·10	·10
16	·31	·09	3	+·07	·13	17	·08	·06
17	·26	·12	4	·08	·19	18	+·03	·11
18	·28	·13	7	·04	·14	20	−·02	...
19	·28	·12	8	·00	·19	21	·05	·07
21	·35	·10	9	·02	·17	23	·15	·05
24	+·38	+·08	10	+·07	+·16	25	−·11	+·02

March 24. Riefler's pressure reduced.
April 1. ,, ,, increased.
14. ,, ,, reduced.
15. ,, temperature found low.
21. ,, ,, ,, high.

Each of these columns supplies material for forming a judgment, but each again is relative to something else, the column

of Riefler's errors being relative to the regular going of that clock, which was interrupted several times by changes in the barometer and temperature, and the Paris references introducing the Paris clock. Up to March 31 these references include a revision of the times originally sent out from Paris, which was kindly communicated by M. Baillaud. If now we take, say, the dates March 10–18, where the azimuths run $^{s}\cdot322$ (W.), $^{s}\cdot220$ (J.S.), $^{s}\cdot329$ (W.), $^{s}\cdot380$ (W.), $^{s}\cdot335$ (W.), $^{s}\cdot245$ (J.S.), $^{s}352$ (W.), and so naturally evoke some suspicion of personality, we are quite unable to trace any corresponding fluctuation in the table last given. Indeed the conclusion from this table would be that the clock errors, and therefore the individual azimuths, were remarkably reliable and did not admit of variation of amount much greater than $\pm 0^{s}\cdot02$. The true nature of the discrepancy between instrumental and stellar azimuths must be reserved for further examination.

Azimuth of North Collimator with respect to South Collimator.

The interest of this is the collateral evidence which it brings to bear upon the degree of fixity of the axis of the South Collimator. A specimen of the curve is shown in $[\alpha]$ in fig. 1. An examination of the fluctuations of this curve, and they are considerable, shows that they follow pretty closely the superficial temperature θ, the coefficient being negative—that is to say, the axis of the North Collimator moving west with respect to that of the South as the temperature rises. There is also some evidence of a daily period apart from this movement. Such a period might perhaps be ascribed to a form of personality, the habit of the observer differing slightly at different hours of the day, or to more rapid temperature fluctuations than are shown in θ, the temperature of the Transit Circle pier. The determination of the temperature coefficient was effected graphically by a method slightly modified from that employed for A and illustrated in fig. 2. A whole or a portion of a month was treated together and one dot registered for the mean of each day. By this means the following coefficients were concluded :—

Temperature Gradient for $[\alpha]$.

Date.	Gradient per 1° F.	Date.	Gradient per 1° F.
	s		s
Oct. 1–16	− ·0040	Mar. 1–31	− ·0027
17–31	38	Apr. 1–10	40
Nov. 1–30	36	11–18	42
Dec. 1–31	36	18–28	35
Jan. 1–31	27	May 1–31	33
Feb. 1–28	− ·0027	June 1–18	− ·0035

The adopted mean coefficient is $-^{s}\cdot0036$ per 1° F. When the weekly means are cleared of superficial changes by the help of this coefficient and reduced to 45°, the basic curve which results is

shown as $[a_0]$ on fig. 3. This curve has already been remarked upon. It has a small and generally steady movement, which shows little trace of following the waves in the temperature θ_0.

Collimation.

There is little indication of the presence of a superficial temperature coefficient in the collimation. Owing to some trouble with the micrometer of the telescope the line of collimation was changed several times during the year, so that it is not possible to draw clear conclusion as to basic changes. The results of collecting the means for periods of about one week are as below:—

Period.		(*c*).
Oct.	1– 4	+ ·376
	5–11	·379
	12–16	·388
	17–19	·221
	20–25	·230
	26–32	·229
Nov.	2– 4	+ ·224
	5– 8	– ·171
	9–15	·162
	16–18	·164
	19–22	·087
	23–29	·079
	30–36	– ·076
Dec.	14–19	+ ·105
	21–27	·112
	28–32	·108
Jan.	2– 8	·114
	12–16	·119
	18–24	·117
	25–31	·115
Feb.	1– 7	·120
	8–10	·120
	15–21	·315
	22–28	·317
Mar.	1– 7	·315
	8–16	·317
	18–21	+ ·306
Mar.	22–28	+ ·298
	29–30	·295
	31–35	·189
Apr.	5–11	·193
	12–18	·193
	20–23	·201
	23–25	·312
	26–32	·309
May	3–10	·313
	10–16	·310
	17–23	·302
	24–30	·303
	31–37	·296
June	7–13	·295
	14–16	·286
	17–20	·272
	21–26	·261
	27–34	·252
July	5–11	·253
	12–18	·250
	19–25	·242
	26–32	·242
Aug.	2– 8	·233
	9–15	·223
	16–22	·211
	23–29	·196
	31–36	·253
Sept.	6–12	·248

The slow steady change between April 23 and August 29 is remarkable.

Perhaps a more interesting feature in *c* is the well-marked

daily coefficient, which is distinct though small, and, like that shown by [α], appears even when there is no corresponding movement apparent in θ. To exhibit it, we collect the observations under the hours 1^h, 7^h, 11^h, and 22^h for prolonged periods; thus we have the means—

1914.	1^h.	7^h.	11^h.	22^h.
	s	s	s	s
Mar. 19–May 21	·306	·304	·300	·300
May 22–July 13	·280	·281	·276	·274

Similarly for nearly the same hours, but with less weight—

1913.				
Oct. 5–Nov. 8	·071	·069	·068	·067

which shows an indication in the same direction. Presumably this is merely a temperature phenomenon, though following changes which are not exhibited by the thermometer of the Transit Circle pier.

Curves of Level.

These were approached in the same way as the curves of azimuth; they were collected first in the same weekly groups and charted, in order to estimate the superficial temperature coefficient graphically. The results are shown in the following table:—

Gradients for Level, Estimated Graphically.

Date.	Range of Temp.	Gradient for 1° F.	Date.	Range of Temp.	Gradient for 1° F.
	°			°	
Oct. 1–5	6·0	+·0126	Feb. 22–28	9·8	+·0076
5–11	5·5	94	Mar. 1–7	6·1	120
12–18	6·5	94	8–14	9·8	93
19–25	18·1	53	15–21	7·3	110
26–32	6·6	60	22–28	4·8	100
Nov. 2–8	9·1	40	29–35	6·8	88
9–15	6·0	102	Apr. 5–11	5·6	92
16–22	9·1	88	12–18	10·5	97
23–29	7·0	93	20–25	14·0	75
30–37	16·3	72	26–32	11·5	61
Dec. 14–20	8·9	88	May 3–9	9·6	95
21–27	10·3	70	10–16	12·4	85
28–32	10·4	104	17–23	7·9	84
Jan. 2–8	11·6	90	24–30	11·5	73
12–16	1·6	...	31–37	10·3	94
18–24	9·8	37	June 7–13	9·3	90
25–31	10·3	68	14–20	7·6	135
Feb. 1–7	6·3	82	21–27	5·1	138
8–14	6·1	98	28–34	10·7	99
15–21	9·7	+·0093	July 5–11	6·7	+·0116

The mean is $+^{s}\cdot0085$ per 1° F. Adopting this as determined, each observation of level was reduced to temperature 45° and the results meaned for each weekly period. The outcome, which is supposed to be free from superficial temperature effects and is called l_0 or the basic level-curve, is shown as the first curve of fig. 3. If the adopted temperature coefficient is in error, the effect would be that of adding to the ordinate of l_0 as shown the due fraction of the corresponding ordinate of θ_0. Its character could not then be changed much, since it already follows θ_0 quite closely in general feature, with reversed sign, though it is decidedly smoother. But its resemblance to A_0 is more remarkable and more interesting. Remembering that the two curves are completely independent, their agreement is as near to identity as one could look for. There is no difficulty in admitting an identity if we suppose that both are due to the same movement of the one V relative to the other, and show merely the vertical and horizontal projections of the same motion. This would imply that the curve A_0 belonged to the Transit Circle alone, and contained no admixture arising from a movement of the South Collimator. If we attribute a portion of A_0 to the latter, it becomes much more difficult to explain the agreement with l_0. The agreement of l_0 with a_0 is much inferior. It seems that we cannot reject A_0 from its claim to represent the true basic azimuth of the instrument unless we are prepared to argue away as without significance the agreement of A_0 with l_0.

In order to determine the temperature coefficient by a method free from possible bias, the weekly groups were treated numerically in the manner that has already been explained in the case of A_0.

Level Gradient, Estimated Numerically, with Mean Residues.

Date.	Gradient per 1° F.	Residue.	Date.	Gradient per 1° F.	Residue.
		s			s
Oct. 5–11	+·0029	±·011	Feb. 1–7	+·0004	±·011
12–18	− 25	·014	8–14	− 20	·012
19–25	+ 40	·012	15–21	+ 27	·013
26–32	29	·009	22–28	+ 11	·011
Nov. 2–8	67	·007	Mar. 1–7	70	·019
9–15	48	·007	8–14	34	·014
16–22	81	·007	15–21	55	·018
23–29	79	·006	22–28	108	·010
30–37	62	·022	29–35	59	·014
Dec. 8–13	...	...	Apr. 5–11	75	·007
14–20	73	·011	12–18	62	·019
21–27	71	·010	20–25	55	·015
28–32	97	·004	26–32	34	·014
Jan. 2–8	+ 85	·009	May 3–9	85	·005
12–16	...	...	10–16	91	·009
18–24	+ 25	·009	17–23	66	·006
25–31	+·0061	±·021	24–30	+·0047	±·013

Level Gradient, Estimated Numerically, with Mean Residues—continued.

Date.	Gradient per 1° F.	Residue.	Date.	Gradient per 1° F.	Residue.
		s			s
May 31–37	+·0064	±·015	Aug. 2–8	+·0062	±·010
June 7–13	97	·011	9–15	63	·012
14–20	37	·016	16–22	72	·012
21–27	147	·009	23–29	56	·013
28–34	53	·012	31–36	47	·010
July 5–11	99	·014	Sept. 6–12	55	·014
12–18	59	·010	13–19	83	·007
19–25	77	·015	20–26	68	·012
26–32	+·0047	±·010	27–33	+·0093	±·013

The general mean gradient is $+^{s}\cdot0057$ per 1° F. This differs from the graphical estimate $+^{s}\cdot0085$ by more than perhaps might have been expected. As a determination it is to be preferred. But none of the conclusions drawn above would be altered by replacing the one by the other.

It is worth mentioning in connection with the above that a basic change of level in the course of the year is observed with the Milne seismograph, which is mounted with its boom North and South. It had been noticed that in the spring the boom required repeated adjustment owing to a tendency to move to the East, with a compensating movement to the West in the autumn, and a hair was attached to it so that the amounts might be measured. The measures, so far as they go, run as follows:—

1913 Sept.	29	0″·0	1914 Jan.	22	5″·6 West.
Oct.	11	0·5 West.	Mar.	29	2·4 West.
	29	1·5	June	22	1·2 East.
Nov.	6	2·1	Aug.	6	2·9
	12	2·1		17	2·5
	17	2·1	Sept.	26	2·1
	20	2·3	Oct.	24	1·2
	22	2·5		26	1·1
Dec.	8	3·4		29	1·0
1914 Jan.	16	5·5 West.	1914 Nov.	2	0·8 East.

That is to say, there is an indicated oscillation of level of amplitude 8″·5 in the course of the season, with maxima about the same season as shown by the Transit Circle, but with phase opposite. The matter is worth sifting further; *prima facie* it may be attributed to a temperature coefficient of the seismograph. For the present the discussion may be closed here. The questions opened possess a good deal of interest and some importance. It is proposed to accumulate further material so that definite and reliable conclusions may be drawn with regard to them.

On Professor Turner's Theory of a Sun-spot Swarm of Meteors, associated with the Leonids. By Professor R. A. Sampson.

Professor Turner's theory of the production of sun-spots by a swarm of meteorites, circulating in an elongated orbit, and refreshed periodically by encounters of Saturn with the Leonid swarm, was published in *Monthly Notices*, lxxiv. 2 (1913 December). In a short work recently issued * I made the following note upon it:—

"More than one attempt has been made to connect the periodicity of the Sun with the planets. That of Professor H. H. Turner is the latest and boldest. He supposes that the sun-spots are produced by meteors detached by encounters of Saturn with the Leonid swarm and projected to the Sun. Certain numerical relations would fit very well with this idea. But it does not appear that Saturn could ever approach the meteors closely enough to produce such an effect—indeed not within a distance about one-half that which separates the Earth from the Sun. Moreover, the perturbations of the planets upon the Leonids are already known. Adams calculated them in 1867, and found that Jupiter produces a regression † of the node of 20′ in 33¼ years, Saturn 7′, and Uranus 1′. These small amounts are known to tally well with observation; and it seems quite certain that nothing so striking as Professor Turner's theory requires can remain to be discovered."

It has been objected to me that it is illegitimate to be quite certain of a negative upon evidence which, as is further objected, is irrelevant. I am not concerned to defend the precise form of a compressed note, inserted upon the proof-sheets of a work which is itself a mere summary, beyond saying that the statements of fact in it are correct, and are clearly distinct from statements of my own judgment upon them. But from another point of view the issue has some general importance. It is undesirable that so novel, attractive, and far-reaching a theory as that of Professor Turner should find a place in astronomical literature without receiving careful examination. Too many instances are known of theories, never really established, which are copied from one work into another upon the credit of a name. I will mention one instance which I shall make good later—Leverrier's theory that the Leonids were captured by Uranus and launched in their present orbit in A.D. 126. I shall show that this often-repeated statement rests upon a very dubious foundation.

Professor Turner's account of his theory is avowedly a rapid sketch. In a note added at the end of his paper he writes, "Several points in the above rapid sketch of a new theory and its possibilities would now be modified; but it is clear that each separate point must be examined with great care before anything like precision could be given to it, and such an examination will

* *The Sun*, Cambridge University Press, 1914.
† *Sic.* It should be "progression."

require much time." One would prefer to postpone criticism till the theory was given in its final draft; but it may facilitate the production of that draft if I indicate some points which seem to me vital to the theory and yet impossible to accept, and some others which are difficult to argue to a positive conclusion, but appear to me not to cohere, and while not vital separately are essential to it for their cumulative effect.

In Professor Turner's theory the SSS, or sun-spot swarm, is a somewhat diffused swarm executing a very elongated orbit which reaches approximately from the Sun to the place where the orbit of the Leonids crosses that of Saturn. The variations of period in recurrence of spots, ranging, according to the data, between 9·4 and 13·3 years, are produced as discontinuities by encounters of the Leonids with the most condensed part of this swarm, near its aphelion, every $33\frac{1}{4}$ years. My first point is that this would require us to attribute to the Leonids an improbable mass.

Consider as a definite problem the encounter of a small Body by a Planet. By encounter I mean a passage so close that for the time being the attraction of the Planet dominates the motion. If S, P are the masses of the Sun and the Planet, A the distance of the latter from the Sun, the distance over which the encounter may be considered to extend may be confined to a distance of the order of d, where $S/A^2 = P/d^2$; for example, for the planet Saturn ($P = 1/3501$) $d = \cdot 16$; for a mass three times that of the Earth, $P = 1/100{,}000$, at the same distance ($A = 9\cdot539$), $d = \cdot030$. These are the distances beyond which the attraction of the Sun would exceed that of the Planet. All that we need regard in the encounter in the way of sensible change of the elements of the Body's orbit we may consider as confined within distances of this order.

Let V be the absolute velocity of the Planet, and v that of the Body relative to the Planet, at the beginning of the encounter. In

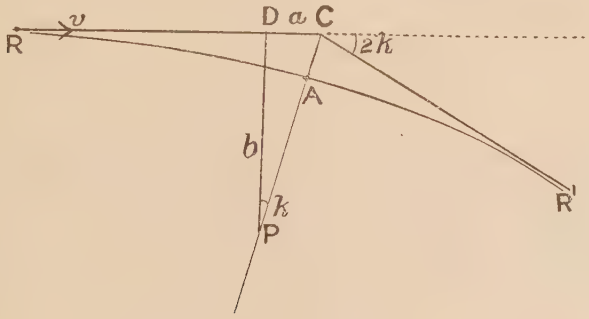

Fig. 1.

almost all cases the encounter will be hyperbolic; the contrary would require $v^2 < 2P/d$; now writing $V^2 = nS/A$, where n will lie between 1 and 2, this will give $v/V < \sqrt{(2PA/nSd)}$, or for the distance contemplated above, $< \sqrt{(2d/nA)}$, that is, a small fraction,

so that the Body when picked up would have to be moving with very nearly the same velocity as the Planet itself. I shall not regard these cases—in which the Planet captures the Body.

In the relative hyperbolic orbit let P be the Planet, at the focus, $PD = b$ the perpendicular upon the original direction of relative motion, which is an asymptote of the hyperbola, $CD = a =$ the semi-major axis, $\kappa = <CPD =$ one-half the acute angle between the asymptotes. We have then the following formulæ, in which the encounter is supposed to be comprised between distances less than r_0 from the focus, before and after which the motion is sensibly asymptotic:—

$$a = b \tan \kappa, \qquad e = \operatorname{cosec} \kappa, \qquad v^2 = P/a,$$

$$r_0 = a(e \operatorname{ch} u_0 - 1),$$

defining u_0.

$$vt_0 = a(e \operatorname{sh} u_0 - u_0),$$

giving $t_0 =$ one half the time of encounter.

$$\Delta\xi = 2b \operatorname{sh} u_0 \cos \kappa,$$
$$\Delta\eta = 2b \operatorname{sh} u_0 \sin \kappa,$$

giving displacements during encounter, along and perpendicular to original line of motion;

$$v \cos 2\kappa, \qquad v \sin 2\kappa,$$

giving velocities after encounter, in same directions.

The last four quantities are relative to P; to find the absolute changes, that is to say what is left after the encounter, when P moves out of range, we must apply the corresponding data for P itself. The pertinent case, and the one to which I shall confine attention, is that in which the Body is at rest when overtaken by the Planet. In this case $v = -V$ and their directions coincide. We then have for the absolute displacements and velocities given to the Body by the encounter:—

$$\Delta x = 2b \operatorname{sh} u_0 \cos \kappa + 2Vt_0,$$
$$\Delta y = 2b \operatorname{sh} u_0 \sin \kappa,$$

and

$$2V \sin^2 \kappa, \qquad 2V \sin \kappa \cos \kappa.$$

The usual case will be that in which κ is a small quantity, for writing $V^2 = nS/A$, we have $a/A = P/nS$, and $\tan \kappa = a/b = (AP/nbS)$. For example, if $P/S = 10^{-5}$, $b = 0{\cdot}1$, $A = 10$, $\tan \kappa < 10^{-3}$. Hence I shall find it sufficient to develop the expressions above to the lowest powers of κ only. We have then

$$a = b\kappa, \qquad e = 1/\kappa,$$
$$r_0 = b(\operatorname{ch} u_0 \cdot \kappa),$$
$$-Vt_0 = b(\operatorname{sh} u_0 - \kappa u_0),$$
$$\Delta x = 2Vt_0 + 2b \operatorname{sh} u_0 = 2b\kappa u_0,$$
$$\Delta y = 2b\kappa \operatorname{sh} u_0,$$

and velocities

$$2V\kappa^2, \qquad 2V\kappa.$$

The x-axis is parallel to the motion of the Planet.

The greatest displacement of position will be less than $2\sqrt{2}\,b\kappa \operatorname{sh} u_0$.

If the Body was found at rest when the Planet arrived, that is to say at aphelion of an orbit of major axis A, it will proceed to describe an orbit of major axis $A + \delta A$, where

$$\delta(v^2) = S\{2/(R + \delta R) - 1/(A + \delta A)\}.$$

Hence since $R = 2A$, for an elongated orbit, in this case,

$$\frac{\delta A}{A} = \frac{\delta R}{2A} + \frac{A}{S}\delta(v^2).$$

By the formulæ above we may disregard the second term, and we shall overestimate the effect if we take $\delta R = 2\sqrt{2}\,b\kappa \operatorname{sh} u_0$. Also if T is the period and δT the increase of it,

$$\frac{\delta T}{T} = \frac{3}{2}\frac{\delta A}{A}.$$

Thus

$$\frac{\delta T}{T} < \frac{3}{2} \cdot \frac{2\sqrt{2}\,b\kappa \operatorname{sh} u_0}{2A},$$

$$< \frac{3\sqrt{2}}{2}\kappa \quad \frac{\sqrt{(r_0^2 - b^2)}}{A};$$

in this r_0 is supposed not to be equal to b.

Now suppose, for example, $r_0 = 5b$, which seems a sufficient allowance of distance over which to follow the encounter, we get

$$\frac{\delta T}{T} < 10{\cdot}6\frac{b\kappa}{A}, \quad < 10{\cdot}6\frac{a}{A}, \quad < 10{\cdot}6\frac{P}{nS}.$$

For the Leonids, at Saturn's distance, $n = 1{\cdot}55$: hence $\frac{\delta T}{T} < 10\frac{P}{S}$; or if we suppose as before $P = 10^{-3} \times S$, that is three times the mass of the Earth, $\delta T/T < 10^{-4}$. Professor Turner's theory would require a change of the order 10^{-1}. To bridge the gap we must increase the mass P.

A second point upon which I would remark is the encounter which the theory requires of Saturn with the Leonids at intervals of 9 revolutions of the former and 8 of the latter, that is to say, about 266 years. By this encounter it is supposed that meteorites are detached from the swarm and left behind, practically at rest, so that they proceed to fall towards the Sun, describing highly elongated orbits of which the point of encounter of Saturn is the aphelion, in periods of about eleven years. It is not easy to grasp the character of this encounter. Professor Turner makes it clear that it is not of the type of that which we have just considered, for this would not leave the detached swarm at rest. He conjectures an actual impact upon the loose structure of Saturn's ring.

I do not want to press the difficulties of this suggestion, though they are considerable, but simply to examine the positions and dates which are actually before us and which must be reconciled with the theory, unless we are to postulate a new orbit altogether.

The orbit of the Leonids and that of Tempel's Comet—the latter derived by Oppolzer,—as given by Adams (*M.N.*, xxvii.), are as follows:—

Leonids.		Tempel's Comet.
33·25 years	(assumed) period	33·18 years
10·3402	mean distance	10·3248
0·9047	eccentricity	0·9054
0·9855	perihelion distance	0·9765
16° 46′	inclination	17° 18′
51° 28′	longitude of node	51° 26′
6° 51′	distance of perihelion from node	9° 2′
retrograde	motion	retrograde

The specification of the last four items has been, as Dr. Crommelin has remarked, a fruitful source of confusion.

They imply that if we proceed 51° in the direct sense from ♈ we reach the descending node of the Comet's orbit, and a further 9° *in the direct sense* along the orbit brings us to perihelion. The movement of the Comet being retrograde, it arrives at perihelion before the node; it has frequently been taken in the opposite way. The elements used by Stoney and Downing ("Perturbations of the Leonids," *Proc. Roy. Soc.*, vol. 64 (1899) p. 403), are vitiated by this error. So also is the diagram in Denning's article (*Observatory*, 1897, May), which is a repetition of the diagram by Miss A. S. Young in *Popular Astronomy*, vol. iv. (1897) p. 498. As this diagram has been used in Prof. Turner's paper, we may mention also that it marks a regressive motion of the perihelion of the orbit of 28°·5 in the interval between A.D. 1866 and A.D. 126, of which, as far as I know, no calculation had at that time been made. It also marks the longitude of Uranus in A.D. 126 at 190° referred to the equinox of 1866, equal to 166° referred to the equinox of its own date; this is 16° too small.

Taking the elements of the Comet for the mean equator of 1866·0 as given by Oppolzer in his *Bahnbestimmung*, ii. p. 427, and converting to the ecliptic, we should now put Adams's data:—

Tempel's Comet. 1866 I.

Π	42°	24′·0	
Ω	231	26·1	mean ecliptic 1866·0
i	162	41·9	
log q	9·98968		
e	·905427		

We shall use these to calculate from as being probably a better determination than the Leonid observations themselves permit. Following the orbit out we find that it makes its closest approach to the orbit of Saturn about longitude 214°·7, with radius vector 9·807 and distance above the ecliptic 0·875. At this place the true anomaly is 206°·5, and the time before the next perihelion passage 3·23 years. The corresponding radius vector for Saturn is 9·805 and distance above the ecliptic 0·416. The least distance separating the two orbits is therefore 0·46.* The date at which Saturn passed this point is 1865 December 16.

If the Leonids keep to the orbit used above, it seems that no encounter of the intimate kind contemplated by Professor Turner's theory can take place. Indeed, with a separation of 0·46 the attraction of Saturn is only one-eighth that of the Sun. If they do not keep to this orbit, this part of the theory is, in Professor Turner's words (§ 30), "mere speculation, and will only be worth attention if an occasion can be identified on which Saturn actually passed through the swarm." The identification supplied depends upon four pieces of collateral evidence: (i) The ancient Chinese records, (ii) an observation by Mr. J. Carpenter in 1863, March 26, (iii) the error of Bouvard's tables of Saturn over the time considered, (iv) the outburst of solar spots in 1870. With regard to (ii) I would point out that at this date Saturn was 30°, or in distance 5 units, from the crossing point of the orbits; hence whatever the phenomenon was which Mr. Carpenter noted, it does not seem to be related to the Leonids we know. With regard to (iii) in my judgment no stress can be laid upon errors of Bouvard's tables of the magnitude in question. In the *Introduction* (p. xi) to his tables he gives the residual errors left by his elements from the observations upon which they are founded. Reducing these from centesimal to sexagesimal seconds, they may be quoted for the period 1800–1820:—

Errors of Bouvard's Tables of Saturn.

1800	+ 2″·6	1807	− 5″·6	1814	+ 0″·9
1801	− 0·6	1808	− 1·9	1815	+ 2·2
1802	− 9·2	1809	+ 2·9	1816	+ 1·8
1803	− 10·4	1810	+ 6·5	1817	+ 4·2
1804	− 8·3	1811	+ 5·2	1818	+ 0·4
1805	− 9·1	1812	+ 6·3	1819	+ 7·9
1806	− 6·8	1813	+ 2·9	1820	+ 10·5

This is a range of very much the same compass as that which Professor Turner noted at a date when the tables had been extrapolated for upwards of sixty years from their epoch, and so might naturally be expected to work away from the observations in

* In an article contributed to the *Times* immediately after the publication of the theory, I overestimated considerably this minimum separation.

irregular ways. But be Bouvard's tables good or bad, the break noted by Professor Turner begins, as he points out himself, in 1863 or 1864. Hence it is open to the same objection as Mr. Carpenter's observation—that whatever it represents, Saturn was at that date very far from the Leonid orbit.

With regard to (iv) the period assigned to the SSS at this epoch is twelve years, so that an outburst of Sun-spots in 1870, when Wolf's numbers rose suddenly from 74 of the previous year to 139, would correspond to some refreshing of the swarm in 1864. To accept this we must make out a connection of Saturn with the swarm at a time when he was upwards of 12°, or more than 2 units, distant from the nearest point of the Leonid orbit. In short the considerations under (ii), (iii), (iv) appear to me to have no bearing upon an encounter between Saturn and the Leonid orbit which we know.

It may be contended that the whole of the foregoing argument takes the data of the Leonid and cometary orbit too strictly and that a great deal of latitude must be allowed for imperfection in the accepted elements, and also for extent of the swarm far beyond anything that we know. With regard to the latter it is far from my wish to cramp a theory which from its nature one can hardly expect to be precise; I wish merely to use existing numbers to point the question—What in fact is the orbit attributed to the swarm that Saturn perturbs, and what is the evidence for its existence? As bearing on the former point, I do not admit that there is any material uncertainty in the adopted data of either orbit. The separation between Oppolzer's elements for the Comet and Adams's for the Leonids probably over-represents the uncertainty. But besides the confirmation of one another which these mutually independent particulars supply, another test has been put upon the reliability of the data in Adams's calculation of the secular motion of the node in the paper already referred to, and its agreement with the value, $+29'$ per revolution, deduced by Professor H. A. Newton from the progressive change of date of the occurrence of the shower, since A.D. 902. The calculation was repeated in 1898 by Abelmann (*Ast. Nach.*, 3516, p. 203), using the Cometary elements. He gives the following results for the movements in one revolution:—

	$\delta\Omega$.	δi.	$\delta\Pi$.	δe.
Jupiter	$+22'\cdot4$	$+48''\cdot2$	$-2'\cdot3$	$+\cdot00006$
Saturn	$+\ 4\cdot6$	$-33\cdot6$	$-1\cdot3$	...
Uranus	$+\ 0\cdot8$	$-57\cdot3$	$+0\cdot1$	...
	$+27'\cdot8$	$-42''\cdot7$	$-3'\cdot5$	$+\cdot00006$

The agreement with observation is satisfactory; but, as far as is shown by other calculations that have been made, it cannot be

secured if we vary the data. There are two such calculations, each partially applicable, viz. Stoney and Downing's paper referred to above (*Proc. R.S.*, vol. 64), in which the data have been inadvertently varied from those used by Adams, and that of Berberich, "Ueber die Störungen der Bahn des Leonidenschwarmes seit 1890" (*Ast. Nach.*, 3526). Each considers the special perturbations of portions of the swarm; and the results show that the variations of the elements are very sensitive to changes in the data. Therefore the agreement with observation of the calculated secular motion of the node appears to me to show that the accepted elements are very fairly reliable. This is what I meant when I said in my note quoted above (p. 82) that nothing so striking as the theory required could remain to be discovered.

There remains the argument (i) from the Chinese records. I am not inclined myself to brush these aside as many would do. But they stand alone, according to the views expressed above, and standing alone they do not seem to me to form a sufficient basis for a theory. Besides, any force they possess depends upon the acceptance of Leverrier's date A.D. 126 for the diversion of the Leonid swarm to its present orbit by the action of Uranus. A perusal of Leverrier's paper (*Comptes Rendus*, t. 64 (1867), p. 93) ought to provoke caution in regard to this. For the encounter to take place it is necessary that the swarm should be near its aphelion and also near its node with the ecliptic, at the time when Uranus passes through the latter. Leverrier had no determination of the position of aphelion, and assumed its approximate neighbourhood to the node. We now know that node and aphelion were separated by 9° in 1866, and there remains the question of their relative movement in the interval of 1740 years. The only calculation of the secular motion of perihelion which we have is that of Abelmann, quoted above, which makes it regrede sidereally at the rate of 3′·5 per revolution. This would make perihelion cross the node in the interval contemplated, separating them by 18° at Leverrier's date; in that case the distance from the Sun to the node would amount to only 13·3 units. Hence if Abelmann's calculation is reliable, Leverrier's identification of the encounter is astray. And it will be noted that any regression of the perihelion (which we should expect with a node progressing) would give the same conclusion. But if we supply the steps of Leverrier's investigation (omitted in his published paper), it becomes clear that there is no need to go back so far as A.D. 126 to find an encounter. The following table is compiled as a guide to the dates which deserve a closer examination. The entries are given for the successive revolutions of the swarm with respect to its ascending node from A.D. 1850 back to A.D. 126. Ω is the associated longitude of the ascending node; $l - \Delta l$ is the mean longitude of Uranus, cleared for simplicity of secular and long period additions, and taken from Newcomb's Tables; both longitudes being referred to mean equinox at the date.

	Date.	Ω.	$l-\Delta l$.		Date.	Ω.	$l-\Delta l$.		Date.	Ω.	$l-\Delta l$.
		°	°			°	°			°	°
1	1850·0	231	29	19	1253·2	214	343	37	656·5	197	297
2			246	20			200	38			155
3			103	21			58	39			12
4	1750·5	228	321	22	1153·8	211	275	40	557·0	194	230
5			178	23			133	41			87
6			36	24			350	42			305
7	1651·1	225	253	25	1054·3	208	208	43	457·5	191	162
8			111	26			65	44			20
9			328	27			283	45			237
10	1551·6	222	186	28	954·8	205	140	46	358·1	188	95
11			43	29			358	47			312
12			261	30			215	48			170
13	1452·2	220	118	31	855·4	203	73	49	258·6	186	27
14			336	32			290	50			245
15			193	33			148	51			102
16	1352·7	217	51	34	755·9	200	5	52	159·1	183	320
17			268	35			223	53	126·0	182	177
18			126	36			80				

The date A.D. 126 is therefore duly indicated, but there are three others which somehow Leverrier overlooked, in which Uranus crossed the node when the swarm was not far from it, viz. at or about A.D. 1222, 1053, and 885. Let us find more exactly the places of the two bodies at these dates.

Newcomb's Tables of Uranus run from A.D. 1600 to 2000, with epoch 1900. For earlier dates it is necessary to extrapolate. For our purpose all inequalities may be disregarded except the equation of the centre and secular equations—with which are included the great inequality due to Neptune. There is, however, a little uncertainty as to how the extrapolation should be made. Over the period of the tables the secular change of the mean longitude varies as the square of the time from epoch and that of eccentricity as the first power of the same. But if the greater part of the change is due to a long inequality of period T, it would be more correct to take the variations as proportional not to x^2 and x, but to $1-\cos(2\pi x/T)$ and $\sin(2\pi x/T)$ respectively. This has been done, taking $T=4220^y$. The difference between the two methods of computation is, however, small, amounting to less than $0^\circ\cdot2$ in the longitude.

I find then with Oppolzer's elements the comet and the secular motions already mentioned

☄ 1866 I.

	A.D. 1222·0.	A.D. 1053·4.	A.D. 884·8.
	°	°	°
Ω	213·1	208·2	203·5
Π	214·6	212·5	210·5
R	19·63	19·17	18·35

where Π is the longitude of aphelion and R the radius vector; at the same dates for Uranus

	♅		
$\theta (=\Omega)$	213°·1	208·2	203·5
r	18·60	18·58	18·56

FIG. 2.—A.D. 1222.

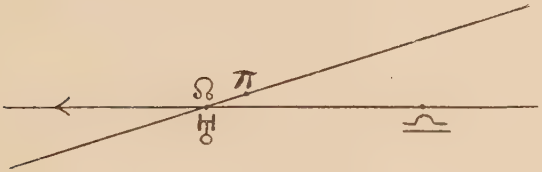

FIG. 3.—A.D. 1053.

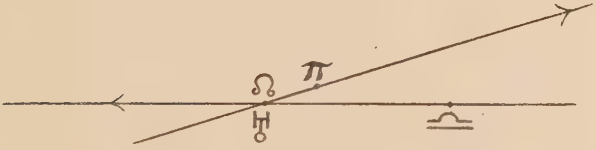

FIG. 4.—A.D. 885.

The relative configurations are shown in the diagrams. The obliquity of the orbit of Uranus has been disregarded, but if included would alter the figures only slightly. There is a considerable separation in radius vector for A.D. 1222, but a correction of $-1°{\cdot}7$ applied to Π in A.D. 1053, or of $+1°$ in A.D. 885, would make the radii exactly equal in one of these cases, but it must be noted that if one is accommodated the other is thrown out.

Carrying forward the ascertained dates with period $33^{y}{\cdot}2$ and comparing them with those of the nearest recorded showers, we have

A.D. 1205	A.D. 1003·6	A.D. 901·4
	1103·2	
cf. (1202·8)	(1002·7)	(902·7)
	(1101·7)	

It will be remembered that the comet 1866 I preceded the meteor showers of the following November by $0^{y}{\cdot}9$.

Treating the first case as undemonstrative, we can conclude that in each of the others there was a very close approach to Uranus. We know that A.D. 1053 is not the date when the swarm was brought into its present orbit, because there are well-attested records of a shower in A.D. 902. But there are none earlier than this, and I can see no objection to the date A.D. 885 for the decisive encounter.

If this is accepted, it is evident that the Leonids cannot be the source of meteors which produce sun-spots; if it is rejected, I do not know, in view of the difficulty as to the position of aphelion, when we shall find a date that is probable.

Note on the Appley Bridge Aerolite of 1914 *October* 13. By W. C. Jenkins, Godlee Observatory, Manchester, and E. L. Rhead, M.Sc. Tech., F.I.C. (Plate 8.)

Having received numerous communications on the appearance of the meteor which caused such widespread alarm in this district on the night of Tuesday, October 13th, at 8-43 p.m., a note was sent to the local press stating its meteoric nature. This brought the information of its discovery from Dr. Wilson of Wigan.

In the interval of correspondence the mass had been removed by the County Police authorities, and it is by the courtesy of the Chief Constable that an examination has been possible.

A comparison with the Catalogue of Meteorites issued by the British (Natural History) Museum indicates that this will rank as the second largest recorded fall in Great Britain.

The dimensions of the mass were:—

Length 9·65 inches
Depth 9·13 ,,
Width 6·62 ,,

with an axial diameter of 10·76 inches.

The weight of the fragments as received by me was 28 lbs. 13 oz., although described by the finders as being of 33 lbs. weight. This may be taken as illustrating the manner in which it had been treated during the week between its fall and reaching the Godlee Observatory.

The form of the aerolite was roughly pyramidal, with more or less a triangular aspect at its lower end. The large end was fractured. Some small portions of material were missing, so that perfect reconstruction was impossible. One detached piece of considerable size accompanied the main body. The fractured surface and also the detached piece exhibited surfaces which were polished almost like slikenside surfaces, and these had a more or less metallic appearance. The latter effect was caused by the quantity of pyrites in the mass, and the surface was the result of the shearing effect

caused by the aerolite striking the ground more or less sideways with considerable force. By this means, the part of the enlarged end that was unsupported by contact with the ground was sheared by the energy of its own impact, with the result stated. These surfaces were not flat, but more or less undulating, so that the actual amount of lateral movement in contact must have been very small. Externally, the aerolite presents all the characteristic appearance of such bodies, and it is in fact a most excellent example of the peculiarities generally exhibited. The exterior of the uninjured portion is covered with the characteristic dark-brown to brown-black skin or cortex, which is quite distinct from the interior. This varies in thickness and in appearance on different parts of the specimen. In some parts it is merely a very thin film, almost a discoloration, and on others attains a thickness of nearly 1·8 mm.

Parts of this skin in the region marked A are quite rough and pimply; others on the lower and narrower end of the pyramid are smoother and smoky in aspect. On the portions which have clearly been well fused, as indicated by their shiny appearance and granulated surface, the effect of the friction produced by the rapid passage of the body through the atmosphere can be clearly traced. The thickness of the coating increases as the base is approached; and at the place marked B, where there has evidently been an irregularity of the surface, the liquid which has been pushed by the friction towards the back of the mass has accumulated as a distinct ridge, giving it almost the appearance of a knife edge which has been melted and is irregular in outline.

Another interesting point is noted just above the ridge referred to. On the fractured surface at the edge of the fracture is a fragment of the outer skin C firmly attached to the surface of the stony fracture, and a little beyond several other small patches can be seen. The first is clearly a detached portion of the outer envelope; the others appear to be splashes only of the fused matter.

The surface of the body presents all the outward appearances usually associated with aerolites. The finely striped and more or less radiately channelled crust and the characteristic pit-like depressions—thumb marks, piezoglyphs—are very clearly marked.

That the origin of all portions of the crust are not simultaneous is shown by the fact that on some surfaces it is limited to a discoloration, and in some cases to a very well-marked tarnishing of the (pyrites) metallic minerals, showing that they had only been heated to a slight extent, probably during the latter part of the flight of the aerolite after the explosion that burst it into fragments, and which was so distinctly heard over a very large area.

From the position of one of these exposed patches near the large end of the mass, it would appear that the production of the pits above referred to might result from the more ready removal by combustion of the sulphur and the oxidation of the iron contained by the pyrites in the mass. The oxide of iron resulting

would, at the temperature of the cortex, unite with the other silicates present in the matrix, producing a very fusible mixture. This would account in some measure for the varying thickness of the outer coating and for the development of the pits. The patch C referred to indicates the extremely liquid condition of the fused outer layer at the moment of fracture, while the chemical composition of the silicates present indicates the low fusibility of the mass as a whole.

The familiar checking and cracking of the crust, due to unequal expansion, is visible on some parts of the surface.

Cracks of a deeper character are also visible, and the whole mass is more or less friable, due to the same cause, to which must be added the shattering effect of the impact of the mass with the earth. It is unlikely that the mass itself was at a high temperature when it struck the ground or that the coating was then molten, as no scoring of the surface such as would result from the friction of the fluid envelope with the ground it penetrated is visible, and it was possible to detect traces of material apparently from vegetable sources on some parts of the specimen. Whether these were acquired when the mass came into contact with the ground or in the subsequent handling, it is now impossible to say. The small depth of penetration is, however, an indication of the low speed of the latter part of the flight, due to retardation by friction—apparently it fell much at the same rate as would a falling body.

The specific gravity of the mass as determined on a detached fragment is 3·336. The specific gravity of the whole mass may slightly differ, but not greatly.

This agrees with the specific gravity that would be anticipated from its chemical and mineralogical composition.

Mineralogical Constitution.—A macrological survey of the material indicates that the main mass of the aerolite consists apparently of olivine and enstatite. There are in addition various other silicate minerals in small quantity, but in the absence of microscopic sections (not yet arrived) it is impossible to identify them. A considerable amount of pyritic matter is also present.

The mineral aspect and also the general form and character of the aerolite under consideration agrees very closely with the larger stones of the Aztec or Holbrook shower of aerolites which fell in Navajo County, Arizona, 1912 July 19, and of which hundreds of small fragments were collected. (See *Am. Jour. Sc.*, ser. iv., vol. xxxiv. pp. 437–456.)

Under a hand lens the surface presents a light ashy-grey colour and a perfectly granulated texture (see photo). The identification of chondrules was not made with certainty. Here and there indications which on microscopic examination may prove to be of this nature were observed, but it was not possible to definitely assert that they were due to this cause. In any case no perfectly formed chondrules were observed.

The olivine present is of a pale yellowish-green colour and is in places well crystallised. The enstatite is whitish or grey or

The Appley Bridge Aerolite, 1914 Oct. 13. Scale $\frac{1}{5}$.

faintly coloured. The fracture displays a considerable amount of pyritic material. Most of this is pyrrhotitic or troilitic (monosulphide) in character and is strongly magnetic. It contains nickel in addition to iron. Some of it is crystalline, and even under a hand lens facets can be clearly seen. It is distributed with a fair amount of uniformity throughout the mass as a whole, but in some parts appeared to form a meshed network in the main body, with here and there distinct aggregations.

In addition to these substances, there is also present a small amount of metallic iron. This is found in very fine particles and in small irregular flakes and granules distributed through the mass. Some of the dust is very fine and some of the granules appear spongy. The largest particles separated did not exceed one mm. in length by half that in width and was in the form of a flat flake with roughened faces. A very marked characteristic is the softness and malleability of the metal. When grinding the small portions used for the analysis, some of the particles rolled up into little cylinders of almost perfect form under the pestle. The iron is very strongly magnetic. It was impossible to estimate the amount of metallic iron in the mass from the small quantity ground up for the analysis, but it is very small. The amount of the pyritical matter can be calculated from the percentage of sulphur found. On this basis it forms 5·06 per cent. of the sample. Nickel is present in notable quantity. It is probably in alloy with the metallic iron and also in the pyrrhotite. In addition, the mass is found to contain small amounts of chlorine as chlorides, insoluble in water, but soluble in nitric acid. The amount is small but distinct. Sodium is a very definite constituent of the mass. Phosphorus is also present in estimable quantities, pointing to the possible presence of schreibersite, the phosphide of iron and nickel often present in aerolites. The amount of lime compounds is small but detectable, and it was thought that a very faint appearance of strontium could be observed. It is somewhat remarkable that the amount of potash present is very small. Traces of antimony and possibly lead were suspected, but the amount of material used was too small to make confirmation of their presence. Professor Knecht informs me that a careful search for titanium gave negative results.

At present no spectrum analysis of the body has been made, but preparations for making it are in progress. The identification of the minerals will be made as soon as the sections arrive. Appended is an analysis of a small portion of the material. As the composition varies in different parts of the mass it only indicates the general character of the body.

The proportions of the minerals worked out on the basis of the composition and solubility, neglecting those occurring in small amounts, would become approximately :—

Pyritic and metallic matter . . .	5·07
Enstatite	31·5
Olivine	63·43

Calculated on the specific gravities of the minerals in question, viz. 4·6, 3·1, and 3·4 respectively, this would give a mean specific gravity of 3·37. The actual specific gravity as determined being 3·336.

Reference has previously been made to the Holbrook stones. The specific gravity of these was nearly the same, namely, 3·22, and a determination on a large portion of the mass of the one now under notice would probably yield a closer approximation.

Mr. W. F. Denning deduced from the various reports of the visible track of the meteor that the motion was from S.S.E. to N.N.W. The position when over Stoke-on-Trent was an altitude of twenty miles, which decreased until it reached the ground at Appley Bridge.

Ultimate analysis as follows:—

Silica (SiO_2)	39·7
Iron (Fe)	20·72
Nickel (Ni)	1·3
Oxygen to combine with excess of Fe and Ni. After deducting sufficient to combine with S present	5·25
Magnesia (MgO)	23·64
Lime (CaO)	traces
Phosphorus (P)	0·23
Sulphur (S)	1·84
Alumina (Al_2O_3)	6·2
Soda, potash, chlorine, etc., by difference	1·12
	100·00

Micrometrical Measures of 110 *Wide Double Stars.*

By W. S. Franks.

The following measures were made with the $6\frac{1}{8}$-inch Cooke refractor at Mr. Frederick J. Hanbury's observatory, Brockhurst, East Grinstead, some particulars of which were given in a former paper (*Monthly Notices*, April 1914). The height above sea-level is about 435 feet. Besides the equatorial there is a $2\frac{3}{4}$-inch Troughton & Simms transit instrument, and a sidereal clock by Frodsham (No. 860), formerly belonging to the late John

Franklin-Adams. Electric light is installed in the observatory, and telephonic connection to the house. Adjoining the observatory is the meteorological enclosure, containing the ordinary equipment of a climatological station, and also including a Campbell-Stokes sunshine recorder. The meteorological readings are regularly published in the local newspaper. The *mean* rainfall here amounts to about 31·3 inches per annum.

Some improvements have recently been effected in connection with the equatorial mounting. (*a*) The balancing of an English equatorial is not very easy, from want of sufficient bearing surface; this, of course, affects the clock driving. A gun-metal ring, in three segments, secured by a projecting arm and steel pillar to the base plate, and working on a sheave rigidly attached to the lower pivot of polar axis, has been added; any desired amount of friction being obtained from the adjusting screws which unite the three segments of ring. (*b*) The slow motion in R.A. was very defective; it has been discarded for a more modern arrangement with coned bearings and adjusting screws to take up the wear (*Monthly Notices*, November 1914). (*c*) The running of the driving clock had always given considerable trouble, due to the defective governor. This has been entirely reconstructed, and the bearing surface reversed. The governor now carries a revolving disc with loose sleeve, working against a fixed screw contact, and capable of very delicate adjustment. This is a great improvement, and the smoothness of motion leaves nothing to be desired; moreover, it is almost noiseless, and only takes 30 lbs. weight, instead of 45 as before, to drive it. All these alterations were skilfully carried out by Mr. J. S. Adams, the Brockhurst mechanician, and they have proved to be successful in facilitating the work of measuring. The position micrometer has also been thoroughly overhauled by Messrs. Cooke & Sons, and several minor defects remedied, with the addition of a new eye-piece of higher power.

The measures were made with the Cooke position micrometer and powers of 235 and 345, the higher power being reserved for the closer pairs and fainter *comites*. Three measures of angle and four of distance were taken on two or more nights. One revolution of screw $= 25''{\cdot}38$. The magnitudes are usually those of the catalogues from whence derived; in a few cases, where I have noticed obvious discrepancies, I have substituted my own estimates, prefixed by Fr. The R.A. and Decl. for epoch 1900 is merely approximate.

It was previously stated (*Monthly Notices*, lxxiv. p. 517) that *duplicates* were excluded from my list of the OΣΣ stars, being elsewhere recorded. There are about 52 of these, out of a total of 256; a list of them, with their synonyms, is here given, for convenience in reference:—

List of the OΣΣ *Duplicates.*

OΣΣ.	
6	=Σ 1, App. I.
10	=Σ 2, App. I.; =Σ 90.
12	=OΣ 24, AB.
14	=OΣ 28 (*comes* of OΣΣ 14).
18	=OΣ 30, AC.
29	=Σ 5, App. I.
32	=Σ 6, App. I.
33	=Σ 373 *rej.*, AC.
34	=Σ 7, App. I.
35	=OΣ 57, AC.
42	=Σ 8, App. I.
43	=OΣ 77, AB-C.
56	=Σ 618.
59	=Σ 12, App. I.
60	=Σ 634.
67	=OΣ 118, AB-C.
76	=OΣ 137 *rej.*, AC.
106	=Σ 1472.

OΣΣ.	
110	=Σ 19, App. I.
113	=Σ 20, App. I.; =Σ 1579, AC.
115	=Σ 1591.
120	=Σ 22, App. I.; =Σ 1678.
128	=Σ 26, App. I.
133	=OΣ 291 *rej.*
134	=Σ 1919.
136	=Σ 27, App. I.
139	=Σ 28, App. I.
147	=Σ 30, App. I.
148	=Σ 31, App. I.
150	=Σ 2111 *rej.*
154	=Σ 2185, AC.
155	=Σ 34, App. I.
156	=Σ 35, App. I.
166	=Σ 2278, AB.
169	=Σ 36, App. I.; =Σ 2323, AC.

OΣΣ.	
173	=Σ 38, App. I.
175	=Σ 39, App. I.
183	=Σ 43, App. I.
184	=Σ 2549, AC.
186	=Σ 44, App. I.
189	=Σ 46, App. I.
193	=Σ 48, App. I.
201	=Σ 49, App. I.; =Σ 2637, AC.
204	=Σ 50, App. I.
218	=Σ 11, App. II.
219	=Σ 56, App. I.
233	=Σ 2900, AC.
235	=Σ 58, App. I.
240	=Σ 2960, AC.
247	=OΣ 496, AB. Double-double, AC-BD.
249	=Σ 3041, A-BC.
250	=OΣ 507, AB-C.

Twenty-eight of the above are common to Σ's Appendix I., given in Section II. (*Monthly Notices*, pp. 527–9); and five others, OΣΣ 60, 106, 184, 218, and 249, appear in Section III. (pp. 530–2). Two objects, OΣΣ 118 and 160, were not found by Dembowski, nor by me; the former is possibly identical with 117. OΣΣ 158 is said to be identical with Σ 2203, on the authority of vol. xl., *Memoirs R.A.S.*; but, as this is a pretty close pair, it seems scarcely probable? Measures of all the remaining duplicates, with their synonyms, are given in the following list.

A few Struvian pairs with proper motion (Section III., p. 530) and some of Herschel's objects (Section IV., p. 532) were held over from the first list—not being then in an accessible position; they are now included here; the remainder are chiefly wide pairs from OΣ, taken from Hussey's revision in vol. v. of the *Lick Observatory Publications*. Σ's Appendix II. has also been completed by the measures of the faint *comites* to Aldebaran, Pollux, η Serpentis, Vega, Altair, and α Andromedæ, all of which have considerable proper motion.

Abbreviations used.—A, angle; D, distance; <, increasing; >, diminishing; "Cel. Obj." refers to Webb's *Celestial Objects*; the column headed "Nights" is the number of nights of observation, and the values in the three previous columns are the *mean* of these.

Measures of Wide Doubles from Struve, Otto Struve, and Herschel.

Syn.	R.A. 1900. h m	Decl. 1900. ° ′	Mags.	Pos. Angle. °	Distance. ″	Epoch. 1914.	Nts.	Remarks.
Σ II 13	0 3·2	+28 32	2·0, 10·7	277·33	75·82	·651	3	α Androm. A and D <.
OΣ 2	,, 8·2	26 26	6·9, 9·6	226·6	17·64	·649	2	AB–C.
,, 6	,, 15·8	66 27	7·2, 9·5	114·55	13·89	·639	2	AB–C = Σ 26 *rej.*
,, 7	,, 16·2	65 55	7·0, 8·0	77·0	51·97	·639	2	A–BC.
,, 23	1 4·2	51 13	7·5, 8·0	191·9	14·56	·658	2	
,, 24	,, 4·6	50 29	7·0, 8·8	68·25	60·12	·658	2	AB = OΣΣ 12.
,, ,,	,, ,,	,, ,,	,, 10·0	286·25	43·50	·658	2	AD.
,, 26	,, 7·5	29 32	6·5, 9·8	258·3	10·75	·658	2	
,, 28	,, 9·7	80 20	Fr. 7·0, 7·0	205·75	130·64	·639	2	A–BC = OΣΣ 14
,, 30	,, 19·9	31 2	7·5, 7·8	105·15	56·95	·658	2	AC = OΣΣ 18.
,, 33	,, 30·8	58 8	7·2, 8·3	75·95	25·67	·669	2	A and D <.
,, 35	,, 37·3	55 22	6·8, 10·0	103·1	11·38	·669	2	A >; D <.
,, 54	3 22·7	67 15	7·2, 8·5	356·6	24·36	·740	2	A <; D >.
,, 56 *rej.*	,, 24·5	47 31	6·5, 10·0	351·45	25·57	·740	2	
,, 57	,, 27·5	23 2	7·0, 7·5	34·65	70·43	·740	2	AC = OΣΣ 35 D >.
,, 68 *rej.*	,, 52·5	47 52	7·0, 8·0	175·25	39·85	·740	2	D <.
,, 77	4 9·6	31 27	7·5, 8·0	41·75	56·59	·289	2	AB–C = OΣΣ 43.
Σ II 2	,, 30·2	16 19	1·0, 10·5	34·23	119·86	·871	3	α Tauri. D <.
Σ 618	,, 54·2	62 55	7·0, 7·3	211·2	33·21	·289	2	= OΣΣ 56.
OΣ 94 *rej.*	,, 59·6	50 10	7·8, 9·5	304·5	18·65	·870	2	AB. D < ? } Triple.
,, ,,	,, ,,	,, ,,	,, 10·0	63·3	26·03	·870	2	AC. D < ? } Triple.
OΣ 117	5 41·8	30 30	7·0, 9·7	29·5	11·70	·904	2	
,, 118	,, 42·4	20 50	6·2, 7·2	161·1	75·34	·289	2	P. V 222. AB–C = OΣΣ 67.
,, 120 *rej.*	,, 47·5	53 26	7·0, 8·0	136·1	45·81	·311	2	D <.
,, 126	,, 53·6	17 49	7·5, 10·0	60·35	10·40	·904	2	
,, 128 *rej.*	,, 56·6	51 35	6·8, 8·3	14·4	38·68	·311	2	35 Cam.
,, 134	6 3·1	24 27	7·0, 8·3	188·55	31·21	·321	2	
,, 137 *rej.*	,, 15·3	21 11	Fr. 7·5, 8·5	257·4	59·20	·319	2	P. VI 62. AC = OΣΣ 76.
,, 146 *rej.*	,, 26·8	11 45	6·0, 9·5	141·0	32·77	·312	2	
,, 147	,, 27·5	38 9	7·0, 8·7	74·05	43·88	·319	2	AB } Quadruple.
,, ,,	,, ,,	,, ,,	,, 9·0	117·25	47·08	·319	2	A–CD } Quadruple.
,, 151 *rej.*	,, 32·6	27 53	7·0, 9·8	134·5	29·43	·910	2	
,, 154	,, 37·3	40 44	6·7, 8·4	120·15	24·11	·312	2	A and D >.
,, 155	,, 39·3	24 47	7·0, 9·9	260·75	15·45	·910	2	
Σ II 5	7 39·2	+28 16	2·0, 10·5	76·95	252·15	·910	2	β Gemin. A and D <.

* OΣ 24. D is rated 10·5 mag., but it is not below 10·0 now.
† OΣ 137 *rej.* 7·7 mag. star in field, at 23°·0 : 265″·16 from A.

Measures of Wide Doubles from Struve, Otto Struve, and Herschel—continued.

Syn.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Dis-tance.	Epoch.	Nts.	Remarks
	h m	° ′		°	″	1914.		
OΣ 188	8 10·1	75 8	6·7, 10·0	193·15	10·46	·910	2	
,, 190 *rej.*	,, 13·9	47 44	7·0, 7·5	280·95	78·51	·313	2	AB. } Triple.
,, ,, ,,	,, ,,	,, ,,	,, 8·5	168·4	38·55	·313	2	AC. }
,, 191 *rej.*	,, 19·0	20 29	7·0, 8·3	192·1	36·96	·313	2	
,, 231 ,,	11 5·6	31 0	7·7, 8·7	264·0	35·88	·326	2	AB.
,, ,, ,,	,, ,,	,, ,,	,, 8·0	334·65	166·59	·326	2	AC. A > ; D <. } Triple.
,, 242 ,,	,, 54·9	71 13	7·0, 7·5	152·1	32·74	·334	2	A < ; D >.
Σ 1591	,, 56·3	0 13	8·0, 8·0	351·8	52·95	·301	2	=OΣΣ 115.
* Hall	,, 58·8	3 55	8·0, 9·0	96·0	112·33	·321	2	A and D <.
OΣ 257	12 52·5	46 9	7·5, 8·2	351·5	13·20	·344	2	
,, 259 *rej.*	13 2·9	24 33	7·5, 8·0	21·35	39·69	·358	2	
,, 262	,, 7·4	74 30	7·3, 8·2	182·1	27·86	·334	2	
,, 274	14 2·4	35 15	7·0, 10·0	65·05	13·83	·603	2	
Hh 457	,, 51·6	−20 56	6·0, 8·0	297·3	18·40	·358	2	P. XIV 212. A and D <.
,, 465	15 6·5	−19 25	5·0, 9·0	111·45	58·62	·358	2	ι Libræ.
,, 467	,, 8·8	−18 3 Fr.	7·0, 8·0	141·65	47·49	·368	2	P. XV 14.
Σ 1919	,, 8·3	+19 39	6·1, 7·0	9·4	24·43	·371	2	=OΣΣ 134.
OΣ 542	,, 18·0	55 41	7·2, 9·8	347·4	11·19	·588	2	
,, 300	,, 35·4	12 23	6·7, 9·8	259·8	15·26	·588	2	
,, 302	,, 51·1	34 39	7·0, 9·0	52·0	28·87	·371	2	
Σ 1993	,, 55·3	17 39	8·2, 8·2	219·4	27·89	·371	2	D > ; A <.
Hh 494	,, 59·6	−19 32	2·0, 4·0	24·9	13·67	·371	2	β Scorpii.
Σ 2007	16 1·3	+13 36	6·5, 8·0	324·55	34·92	·376	2	A > ; D <.
Σ 2010	,, 3·6	17 19	5·0, 6·0	10·6	29·31	·376	2	κ Herculis.
Hh 497	,, 6·2	−19 12	4·1, 7·0	336·8	40·99	·374	2	ν Scorpii. AB–CD.
Σ 2017	,, 7·5	+14 48	7·7, 8·4	253·9	27·73	·376	2	D <.
"Cel. Obj."	,, 31·0	17 15 Fr.	6·5, 7·0	359·4	156·21	·376	2	P. XVI 125–6.
OΣ 316 *rej.*	,, 47·3	59 40	7·0, 8·0	349·35	47·24	·382	2	
Σ 3127	17 10·9	24 57	3·0, 8·1	195·45	12·53	·382	2	δ Herc. A < ; D >.
Hh 534	,, 11·9	−24 11	5·5, 6·0	357·0	12·40	·395	2	39 Ophiuchi.
,, 535	,, 15·2	−12 45	4·0, 8·5	30·6	47·30	·392	2	ν Serpentis.
OΣ 329 *rej.*	,, 21·0	+37 2	5·8, 8·5	12·85	33·18	·392	2	
Σ 2183 *rej.*	,, 30·1	− 5 49	6·8, 8·8	14·3	27·80	·392	2	A < ; D >.
† Σ 2185	17 29·9	+ 6 6	7·0, 10·0	5·05	27·98	·387	2	AB.
,,	,, ,,	,, ,,	,, 7·7	209·4	83·02	·387	2	AC=OΣΣ 154. A <; D >. } Triple.

* Ll 22662 and 22667. Proper motion noted by Hall, in Washington Obs., 1888, App. I.
† Σ 2185. B seems brighter than 10·0 mag. ; more like 9·5 ?

Measures of Wide Doubles from Struve, Otto Struve, and Herschel—continued.

Syn.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″	1914.		
OΣ 332	17 30·4	15 23	7·2, 10·3	110·95	10·46	·606	2	
,, 340	,, 32·3	86 57	7·8, 8·3	229·3	31·50	·551	2	A >, due to precession?
,, 334	,, 40·0	34 49	7·4, 8·8	355·9	15·16	·414	2	
,, 335	,, 41·7	21 56	7·3, 8·3	140·05	25·12	·414	2	
Σ 2278	18 1·1	56 26	6·8, 7·3	24·35	37·68	·404	2	AB = OΣΣ 166. A <; D >. } Triple.
,,	,, ,,	,, ,,	7·3, 7·8	147·65	6·15	·404	2	BC. } Triple.
Σ II 8	,, 16·1	− 2 55	3·5, 11·0	60·93	171·17	·582	3	η Serp. A >; D <.
OΣ 352 *rej.*	,, 23·6	+46 45	7·1, 8·3	222·5	24·30	·560	2	
Σ 2330	,, 26·6	13 6	7·3, 9·0	171·5	18·11	·540	2	A and D >.
OΣ 355 *rej.*	,, 28·6	8 12	6·5, 9·5	248·55	38·67	·554	2	
Σ 2342	,, 30·7	4 51	5·7, 8·5	5·35	30·05	·540	2	A >; D <.
* OΣ 356 *rej.*	,, 30·0	40 5	7·0, 8·8	305·65	35·18	·560	2	AB.
,, ,,	,, ,,	,, ,,	,, 9·5	4·9	50·88	·560	2	AC. A is moving? } Triple.
Σ 2346	,, 32·4	7 27	7·5, 9·0	292·4	22·33	·540	2	A and D <.
Σ II 9	,, 33·6	38 41	1·0, 10·5	166·56	55·0	·493	5	α Lyræ. A and D <.
OΣ 361	,, 38·7	5 32	7·5, 8·2	173·0	22·47	·554	2	
,, 366 *rej.*	19 10·5	34 3	7·2, 9·8	230·6	21·58	·596	2	
,, 367 ,,	,, 10·8	34 23	6·8, 9·3	227·65	33·53	·565	2	A–BC.
,, 370	,, 12·3	9 10	7·5, 8·2	14·0	19·67	·565	2	P. XIX 49.
,, 379 *rej.*	,, 34·2	33 40	7·2, 8·5	86·4	24·58	·572	2	
Σ II 10	,, 45·9	8 36	1·5, 10·2	303·43	162·39	·557	3	α Aquilæ. A >; D <.
OΣ 389	,, 48·7	30 53	6·9, 8·8	183·15	12·59	·572	2	
,, 391	,, 54·0	44 0	7·5, 10·2	51·35	18·46	·606	2	
,, 393	,, 54·7	44 7	7·5, 8·4	227·75	19·28	·572	2	A <; D >.
,, 394	,, 56·6	36 8	7·0, 9·8	294·5	11·00	·595	2	
,, 396 *rej.*	,, 58·8	18 13	6·0, 9·5	205·85	47·35	·572	2	
,, 397 ,,	20 0·2	15 37	6·5, 8·2	174·05	38·77	·592	2	A and D <.
,, 404 ,,	,, 13·1	52 12	7·0, 9·5	114·85	29·79	·592	2	
,, 411	,, 39·0	45 28	7·4, 10·2	321·9	18·11	·606	2	A and D <.
,, 414	,, 43·5	42 3	7·2, 8·3	94·45	10·05	·604	2	
,, 421 *rej.*	,, 51·8	31 43	7·3, 9·5	192·95	36·73	·604	2	
,, 428 ,,	21 4·1	+ 6 20	7·8, 9·3	256·2	24·68	·604	2	
,, 433	,, 13·8	34 29	Fr.4·5, 10·3	219·8	14·88	·610	2	AB υ Cygni.
,, ,,	,, ,,	,,	,, 10·1	179·8	21·44	·610	2	AC (C brighter than B). } Triple.
,, 434 *rej.*	,, 15·1	39 20	6·7, 9·5	121·65	24·68	·612	2	
,, 450 ,,	,, 41·5	6 4	7·2, 10·0	248·7	42·23	·612	2	

* OΣ 356 *rej.* Hussey has transposed the measures of AC and BC in vol. v., *Lick. Obs. Pub.*

Measures of Wide Doubles from Struve, Otto Struve, and Herschel—continued.

Syn.	R.A. 1900.	Decl. 1900.	Mags.	Pos. Angle.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″	1914.		
OΣ 459	21 56·7	39 3	7·5, 10·2	194·9	10·78	·612	2	
,, 467	22 10·1	22 2	6·3, 10·3	273·5	23·00	·618	2	
,, 469	,, 16·1	34 37	7·2, 8·8	285·5	29·95	·618	2	A <; D>.
Σ 2900	,, 18·8	20 20	6·0, 7·9	321·45	69·29	·618	2	33 Peg. AC=OΣΣ 233. A >; D <.
OΣ 473	,, 26·5	56 43	6·7, 10·0	357·1	14·78	·622	2	
,, 480	,, 42·1	57 33	7·5, 8·2	116·9	30·90	·622	2	
Σ 2960	,, 51·7	41 4	6·0, 9·0	47·4	62·18	·618	2	16 Lac. AC=OΣΣ 240.
OΣ 485 *rej.*	,, 58·4	54 42	6·0, 9·0	50·4	20·50	·625	2	A and D >.
,, 486 ,,	,, 59·3	59 54	6·2, 8·8	276·25	34·12	·625	2	
,, 496	23 25·4	58 0	5·4, 7·4	269·35	75·66	·625	2	P. XXIII 100-1. AB =OΣΣ 247.
,, 498 *rej.*	,, 26·6	51 52	7·2, 10·0	243·05	17·50	·625	2	
,, 499	,, 28·5	56 51	7·2, 8·8	78·9	9·67	·627	2	
,, 507	,, 43·8	64 19	6·8, 7·8	352·0	49·74	·629	2	AB–C=OΣΣ 250.

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